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Optimizing sustainable industry investment selection: A golden cut-enhanced quantum spherical fuzzy decision-making approach

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HIGHLIGHTS

- Sustainable industry alternatives in emerging markets are ranked based on ESG index.
- The golden cut-enhanced quantum spherical fuzzy decision-making approach is used.
- H_2O Emissions, Innovation, Community Investment found to be the influencing factors.
- Innovation came out to be the strongest ESG performance criterion.
- Technology and communication are found to the best-performing industries.

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ABSTRACT

This study aims to rank sustainable industry alternatives in emerging markets based on the directional impact relations of the environmental, social, and governance (ESG) index components for a socially and environmentally conscious investment strategy. To achieve this goal, we employ a golden cut-enhanced quantum spherical fuzzy decision-making approach. Specifically, we first use a quantum spherical fuzzy DEMATEL technique to identify the impact-relation directions and the weights of the ESG criteria set. Second, we employ the extended TOPSIS with the quantum spherical fuzzy sets to rank the industry alternatives concerning their directional ESG performances. The findings show that (i) H_20 Emissions, Innovation, Community Investment, Gender Equity, Human Rights, and CSR Strategy are the main influencing factors based on their impact-relations directional scores, (ii) Resource Usage, Product Responsibility, and Shareholders' Rights are the set of criteria under the influence of remaining ESG, (iii) Innovation is the strongest ESG performance criterion, whereas Human Rights is the weakest, (iv) technology and communication are the best-performing industries based on the directional ESG index performance scores, whereas real estate and basic materials industries are the worst performing. The study provides valuable and actionable insights for companies that aim to make socially responsible investments.

1. Introduction

Many companies have been incorporating sustainability matters in their strategies and business processes by reducing the release of harmful substances to the flora ecosystem [1,2]. A similar shift has taken place in sustainable finance and investors have been remarkably

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inclined to make investments that yield benefits concerning environmental, social, and governance (ESG) performance [3–5]. Institutional investors usually take investment decisions based on the companies' successful engagement in ESG matters measured by parameters. Thomson Reuters (TR) ESG Index is one of them and it measures the ESG scores of companies across the globe [6]. There are 3 pillars and 10 subproxies of the TR ESG Index. The environmental pillar has 3 components, i.e., H20 Emission, Resource Usage, and Innovation, the social pillar has 4 components, i.e., Community Investment, Gender Equity, Product Responsibility, and Human Rights, and governance has 3 components, i.e., Shareholders' Right, Management Structure and Compensation and CSR Strategy.

There are more than 3000 institutional investors that signed the "Principles for Responsible Investment" agreement to contribute to sustainability [7]. Due to the concerns raised by corporate scandals over the last two decades, stakeholders and non-governmental organizations (NGOs) are keen on pushing companies to take measures towards complying with ESG standards and industry regulations ever than before [8–10]. Hence, companies are expected to disclose adequate information about their commitment to the development of sustainability capabilities to boost corporate reputation [11–13]. Moreover, with the continuous degradation of ecosystems and depletion of the ozone layer, there is a high call for companies, particularly those in the manufacturing industry, to incorporate sustainability actions in the production processes to protect the environment [14]. This is not surprising since the activities of these companies cause significant consumption of resources and release harmful substances into the environment. Thus, they should adopt green production techniques as the highest priority [15].

More so, in channeling their funds in different investment portfolios, investors rely on the sustainability performances of firms as a common denominator to their investment decisions. Besides, with more attention to responsible investment by investors research in this domain has been rising observing the performance of green stocks and assets in the market. Most recently, studies have attempted to examine the relations between listed firms' ESG performance and institutional investors' willingness of holding more shares [16,17]. Besides, [18] conducted a study on the impact of ESG performance on bond credit risk with studies conducted by [19] examining the effects of environmental, social, and governance scores of firms on shareholders' returns. More on the debate of ESG funds, [20] have investigated the equity performance of European ESG funds in the energy sector. Similarly, studies have been mounted on emerging markets investors' socially responsible actions based on the random walk approach [21], and investment dynamics of eco-friendly stocks about cross-asset performances [22].

However, there is still a gap in the literature in evaluating the impact of the ESG performance of industry alternatives on investment decisions. The extant literature mainly employs traditional methods and econometric tools to analyze the potential nexus between ESG variables and the corporate performance of companies. There are relatively few studies that examine the weight of the ESG Index components and the possible impact-relation directions among them to shed light on the priority ranking of these components using Multicriteria Decision Making Methods in a fuzzy environment. There is also a need to develop a holistic approach to the sustainability of industry alternatives for long-term investment decisions as a trade-off between being socially responsible and having good corporate performance.

Another important gap in the literature is that there is a strong need for generating specific strategies for the investors to provide socially and environmentally improvements. However, all improvements of the investors regarding this issue create new costs. In other words, making lots of improvements related to the ESG components are quite difficult for the investors due to the budget constraints. Therefore, there is a need to make a priority evaluation regarding these indicators. In the literature, most of the scholars focused on the significance of these indicators. However, there are limited studies in the literature that focused on the most essential items. Because of this situation, a priority evaluation can help to identify critical factors in this regard. With the help of this issue, these investors can implement their strategies without facing high costs.

In this study, our proposed methodology marries the principles of quantum mechanics and fuzzy sets, forging a powerful decision-making framework that takes advantage of the phase angle's significance for the ESG components. By incorporating the amplitude and phase angles, our model achieves improved accuracy and enhanced understanding of uncertainty, making it widely applicable across diverse domains. The simultaneous evaluation of membership, non-membership, and hesitancy degrees allows for a holistic understanding of uncertain information, guiding informed decision-making based on multiple parameters [23,24]. The pivotal role of phase angles in characterizing complex systems and their applications in various academic disciplines underscores the importance of integrating them into our model. Leveraging the amplitude and phase angle items within quantum theory, our model presents a profound outlook on probabilities, yielding innovative solutions to real-world problems. The quantum model of mass function with diverse angles provides a precise examination of uncertainty in complex information sets governed by quantum logic [25,26].

Moreover, quantum spherical fuzzy sets, incorporating the phase angle as a pivotal parameter, enrich the analysis of complex information sets and decision-making problems. The unique representation of quantum fuzzy numbers with phase angles provides valuable insights into relative positioning and phase relationships, enabling a more accurate representation of uncertainties. However, defining precise membership, non-membership, and hesitancy degrees within spherical fuzzy sets remains an outstanding academic pursuit. To address this challenge, the proposition of the golden cut, drawing from ancient mathematical principles, offers potential solutions by uncovering optimal ratios among the scales of spherical fuzzy sets for our business analytics approach to ESG performance.

Accordingly, this study aims to develop a business analytics approach to the selection problem of investors on sustainable industry alternatives based on the ESG performance in emerging markets. We postulate the following research questions; (i) how to evaluate the relative importance and weight of the TR ESG Index components, (ii) how to figure out the impact-relation directions of ESG Index components, (iii) and how to rank the industry alternatives based on the ESG performance for sustainable investment strategies. We employed the golden cut-enhanced quantum spherical fuzzy decision-making approach that uses Fuzzy DEMATEL and extended TOPSIS methods to rank the industry alternatives in emerging markets. In our proposed hybrid model, we first used a quantum spherical fuzzy DEMATEL technique to figure out the impact-relation directions and the weights of the ESG criteria set. Our set of criteria is the TR ESG Index with its 10 sub-proxies. Second, we employed the extended TOPSIS with the quantum spherical fuzzy sets to rank the industry alternatives for their directional ESG performances based on their impact relations. In our model, based on the extensive literature review and expert opinions, we selected ten industries from a popular research engine: Financial Services (A1), Consumer Defensive (A2), Technology (A3), Real Estate (A4), Energy (A5), Communication Services (A6), Healthcare (A7), Consumer Cyclicals (A8), Basic Materials (A9) and Utilities (A10).

The main contributions of this study are shown as follows.

(i) The main novelty of this study is that it crafted a new hybrid model based on quantum spherical fuzzy sets using a two-stage decision-making approach, i.e., quantum spherical fuzzy DEMATEL for the impact-relation directions and the weights of the ESG Index components as a set of decision criteria, and the extended TOPSIS with the quantum spherical fuzzy sets for ranking the industry alternatives for their directional ESG Index performances. We expect that the ranking of the industry alternatives will guide socially responsive investors through the employment of multi-criteria decision-making techniques in making the right investment choices.

(ii) Using DEMATEL technique to weight the indicators provides many advantages. The main difference of DEMATEL by comparing other similar approaches is that it can consider causal directions while weighting the items [27,28]. The criteria for the ESG Index may have an influence on each other. As an example, decreasing the level of greenhouse gas can also be very helpful to increase the corporate social responsibility activities. Therefore, with the aim of making appropriate evaluation, the causality situation should be taken into consideration. Due to this issue, DEMATEL is the optimal methodology in this framework [29]. In other words, the proposed model has essential superiorities in comparison with the previous ones in which other techniques were used [30–32].

(iii) Integration of quantum theory with Spherical fuzzy sets also increases the quality of the proposed model. Quantum theory can consider different probabilities in the analysis process. This situation provides an opportunity to make more effective evaluations in complex conditions. Moreover, Spherical fuzzy sets take into consideration a wider range of data in the analysis process. This situation helps to reach more appropriate findings. Finding the most significant criteria for the ESG index and identifying the most critical sector for the ESG performance are very crucial issues. Hence, to make effective evaluations, different probabilities should be considered. Owing to this situation, a complex model should be created such as integrating Spherical fuzzy sets with Quantum theory.

(iv) Besides, employing these techniques provides clarity in the literature on how criteria for each ESG indicator influence other indicators. Specifically, evidence is provided on the influencing roles among the criteria of the ESG indicators which provide a guide to investors' responsible investment decisions [33]. Furthermore, following the rising debate on sustainable investment, most of the extant research was focused on traded securities [16,18,19,22] and financial returns of firms, leaving a gap in the body of knowledge on the effects of non-financial indicators on investors' responsible investment initiatives. Thus, by simultaneously examining all three components of the ESG indexes, the study aims to enrich the literature by providing insights on the attractiveness of an industry based on the lens of sustainability and non-financial performances of firms that could attract the interest of potential investors. Most importantly, the results help simplify investors' decision-making process on which industry to invest in, thus, minimizing the risk involved in engaging in irresponsible investments which might have spill over effects on corporate image and legitimacy.

The remaining manuscript is organized as follows: The following section reviews the relevant literature and provides theoretical background. Section 3 provides the methodology and the proposed decision-making hierarchy of investors. Section 4 discusses the results of the analysis. Finally, the last section concludes the paper by discussing its implications and providing future research directions.

2. Theoretical framework and relevant literature

2.1. Theoretical framework and literature review

The debate on responsible investments and companies' sustainability performance has been in the interest of many theories. Stakeholder theory together with agency theory provides the most prominent insights into how companies perform in their relationship with shareholders in terms of value maximization through sustainability. The stakeholder theory recognizes the significance of managers clearly defining business boundaries and relationships with shareholders in capturing corporate gains [34]. In this respect, the efforts of managers in ensuring value creation from the perspective of ESG matters have paramount importance.

With the agency theory, the actions of managers are monitored to prevent any conflict of interest between managers and shareholders [35]. Managers are mostly responsible for securing corporate resources, and thus, monitoring them helps ensure that they take responsible actions in line with the sustainable development goals without affecting corporate reputation [36]. As noted by Suttipun [37], the agency theory is also a viable tool for revealing the nature of corporate ESG disclosures. Moreover, in conjunction with these theories, the legitimacy and media agenda-setting theories play a significant role in communicating the behavior of companies to society in terms of corporate sustainability practices [38]. Through social contracts, these practices may boost the legitimacy and reputation of the companies [39,40].

Closely related to the preceding discussions, managers are accounted responsible for providing non-financial information to investors [41,42]. This is especially the case when investors prefer investing in ESG funds that include sustainability-conscious companies [43,44]. In this vein, ESG facets are considered as effective means of communicating with stakeholders [8,45,46]. Thus, as argued by D'Amato et al. [9], the financial performance of companies highly depends on their transparency with stakeholders in terms of sustainability practices. The same outcome is valid for boosting corporate reputation [44,47]. Further, the sustainability practices of companies enable them to earn a competitive advantage in the industry due to the reduced level of carbon emissions and pollution [12,48,49].

Several studies have attempted to examine the relationship between corporate sustainability performance and financial performance [50–58]. Using Ohlson's O-score, Lisin et al. [59] examined the influence of ESG pillars on investment return and found a positive relationship. Similarly, Abdi et al. [60] indicates a positive influence of the sustainability practices of airlines on Tobin's q. In a similar vein, Buallay et al. [61] indicates a significant positive relationship between the ESG scores of companies and investment return. Some studies have shown ESG scores of companies to be an important determinant of the

Table 1
Selected criteria for the ESG Index.

Dimensions	Criteria	Explanation
Environmental (D1)	H2O emission (C1)	The level of greenhouse gas release in the atmosphere affects the environment and the ozone layer
	Resource usage (C2)	The level of energy and sustainable packaging practices of companies
	Innovation (C3)	The use of new technology and processes in the production and operational activities
Social (D2)	Community investment (C4)	The corporate social responsibility of the companies in relation to what is given back to the community
	Gender equity (C5)	The recognition of gender equality and zero tolerance for discrimination in the working environment
	Product responsibility (C6)	The production and marketing of quality products that are free from any hazard
	Human rights (C7)	The level of human rights for effective organizational behavior and the convenience of the fundamental conventions
Governance (D3)	Shareholders' rights (C8)	The voice and rights of shareholders in the management and operation of the company
	Management structure and compensation (C9)	Board members' level of diversity and the control system of the company
	Corporate social responsibility (CSR) Strategy (C10)	Transparency in the CSR reporting of companies

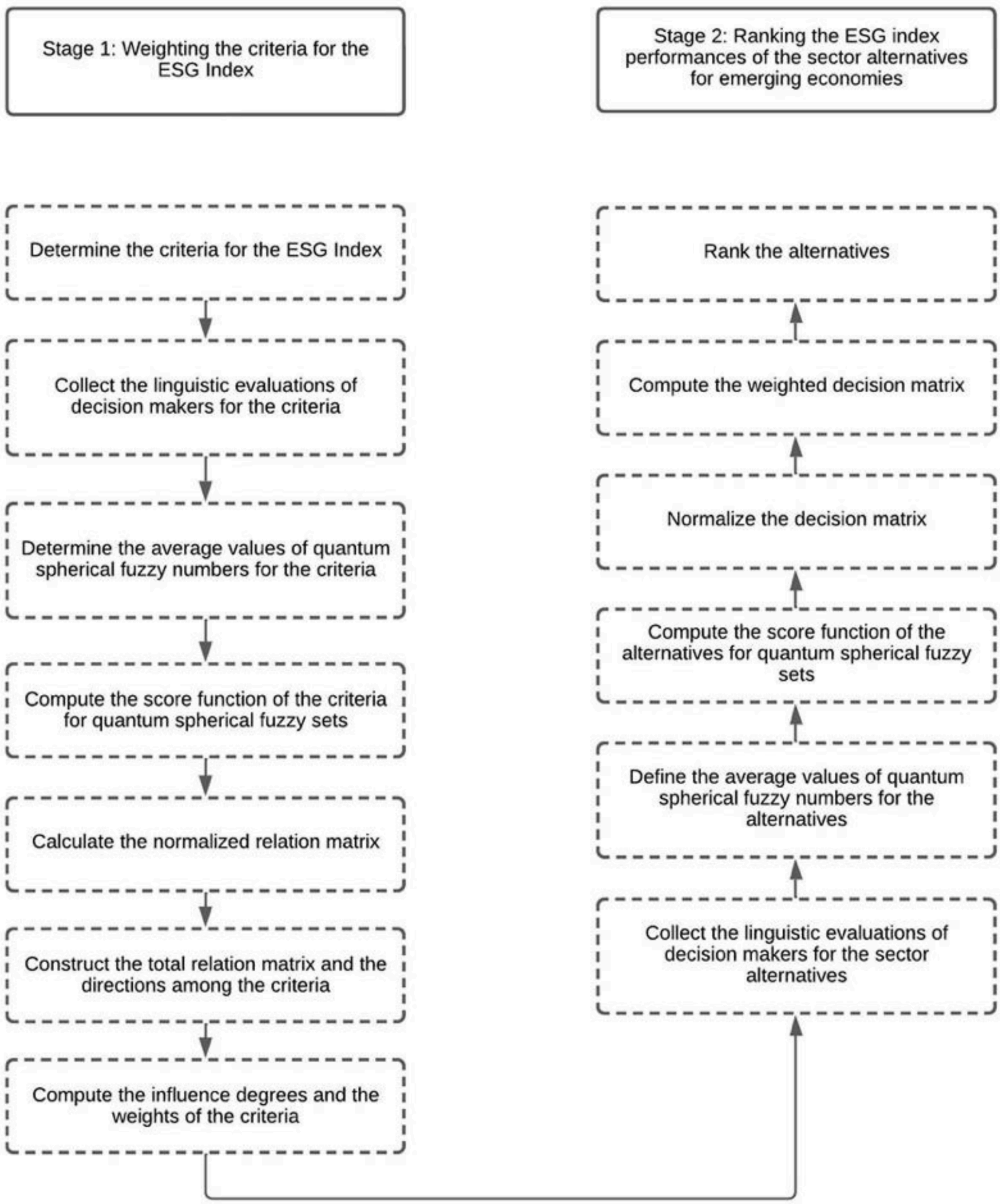

Fig. 1. The flowchart of the proposed model.

Table 2
Linguistic scales and golden cut-based quantum spherical fuzzy numbers.

Linguistic scales for criteria	Linguistic scales for alternatives	Possibility degrees	QSFNs
No influence (n)	Weakest (w)	0.40	$\left[\sqrt{0.16}e^{j2\pi.0.4}, \sqrt{0.10}e^{j2\pi.0.25}, \sqrt{0.74}e^{j2\pi.0.35}\right]$
Somewhat influence (s)	Poor (p)	0.45	$\left[\sqrt{0.20}e^{j2\pi.0.45}, \sqrt{0.13}e^{j2\pi.0.28}, \sqrt{0.67}e^{j2\pi.0.27}\right]$
Medium influence (m)	Fair (f)	0.50	$\left[\sqrt{0.25}e^{j2\pi.0.50}, \sqrt{0.15}e^{j2\pi.0.31}, \sqrt{0.60}e^{j2\pi.0.19}\right]$
High influence (h)	Good (g)	0.55	$\left[\sqrt{0.30}e^{j2\pi.0.55}, \sqrt{0.19}e^{j2\pi.0.34}, \sqrt{0.51}e^{j2\pi.0.11}\right]$
Very high influence (vh)	Best (b)	0.60	$\left[\sqrt{0.36}e^{j2\pi.0.6}, \sqrt{0.22}e^{j2\pi.0.37}, \sqrt{0.42}e^{j2\pi.0.03}\right]$

Table 3
Linguistic evaluations of decision makers for the criteria.

Decision maker 1										
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
C1		M	M	H	H	S	VH	S	M	VH
C2	S		S	VH	VH	H	S	M	S	VH
C3	M	M		H	S	VH	H	S	VH	S
C4	M	M	VH		M	S	VH	M	VH	VH
C5	S	H	S	VH		VH	VH	VH	S	M
C6	M	M	H	S	S		S	H	S	M
C7	H	H	VH	M	M	H		S	VH	M
C8	VH	M	H	H	H	H	S		S	S
C9	M	H	S	H	VH	S	VH	S		VH
C10	H	VH	M	S	S	H	S	VH	S	
Decision maker 2										
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
C1		M	VH	H	VH	VH	VH	S	VH	VH
C2	S		H	VH	H	H	S	M	H	VH
C3	S	VH		H	S	VH	H	VH	VH	VH
C4	M	M	VH		M	VH	VH	VH	VH	H
C5	S	S	VH	VH		VH	VH	VH	VH	M
C6	S	VH	H	S	S		S	H	H	VH
C7	H	S	VH	VH	M	H		S	VH	H
C8	VH	M	H	H	H	H	S		S	S
C9	M	S	VH	H	VH	S	VH	S		VH
C10	S	VH	M	S	S	H	S	VH	S	
Decision maker 3										
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
C1		VH	S	M	H	M	VH	S	M	VH
C2	S		S	M	M	M	M	M	S	VH
C3	M	M		H	VH	S	M	S	VH	S
C4	M	M	VH		M	S	VH	M	M	VH
C5	M	H	M	H		VH	VH	VH	S	M
C6	M	S	H	S	S		S	M	S	M
C7	H	VH	VH	M	M	H		M	VH	M
C8	VH	S	M	H	S	H	S		S	S
C9	M	VH	S	H	VH	S	VH	S		VH
C10	H	VH	M	H	S	M	H	M	H	

creditworthiness of companies [11]. Atif and Ali [62] show an inverse relationship between firms' default risk and their ESG disclosures. This result is also supported by the findings of Pisani and Russo [4] which indicate a negative relationship between ESG ratings of funds and their risk factors, showing that funds with higher ESG ratings have lower risk levels.

2.2. Literature review in emerging markets

Although there are many studies held on the relationship between ESG performance and corporate performance, there is still a gap in the potential nexus between ESG performance and the sustainability of companies from different sectors in emerging markets. According to Bahadori et al. [63], most of the extant literature on the ESG performances of companies has focused on developed markets, leaving a gap in emerging economies. Few studies that explore the relationship between ESG performance and corporate performance in emerging markets indicate the role of effective management structure (C9), and CSR strategy (C10) on corporate performance, particularly in Southern Asia (i.e., Malaysia, Philippines, Thailand, Indonesia, and India) and Confucian Asian (i.e., South Korea and Taiwan).

Husted and de Sousa-Filho [64] show the positive effect of management structure (C9) on ESG disclosure in Latin America. This result is in line with the findings of the other prior studies held on emerging markets [65–67]. Duque-Grisales and Aguilera-Caracuel [68] examine the linkage between ESG practices and the financial performance of 104 multinationals from Brazil, Chile, Colombia, Mexico, and Peru from 2011 to 2015, and show a significant negative relationship between them. The study also verifies the mediocre effect of ESG performance on the financial services sector (A1). Yoon et al. [69] examine the relationship between ESG performance and corporate value in Korea and show a significant positive relationship between CSR strategy (C10) and firm value.

Other studies examining the relationship between management structure (C9) and ESG disclosures in the Middle East (i.e., Turkey, Egypt, Qatar, UAE, Saudi Arabia) reveal that companies with more diversified board members have more impacts on the sustainability performance of companies. Gender equity (C5) and board diversity enhance business performance with increased ESG scores. Arayssi et al. [70] scrutinize the impact of board composition on sustainability in the Gulf Countries by analyzing the sustainability performance with ESG scores and corporate governance (C8, C9, and C10) for 10 years through applying multiple panel data regressions and sensitivity testing. The results show that companies appointing a sustainability or governance committee tend to execute more social (C4, C5, C6, C7, C8) and environmental activities (C1, C2, C3).

Berezinets et al. [71] investigate a similar relationship between board structure (C9) and corporate performance in Russia. Rajesh [48] investigates the sustainability performance of 39 Indian companies from 2014 to 2018 using environmental (D1), social (D2), and corporate governance (D3) scores, from the TR and found that shareholders score (C8), management score (C9), and human rights score (C7) are among the least influencing indicators. In contrast, the shareholders' score (C8) and management score (C9) affect governance performance. Rajesh and Rajendran [72] study the ESG scores of 820 firms in emerging markets for five consecutive years and observe a significant negative moderating effect of ESG performance, independently over the all-direct relations. Jyoti and Khanna [73] examine the environmental scores of Indian companies and found a positive relationship with their ROA, while their social scores indicate a reverse relationship.

Almost all these studies show that ESG performance through good governance practices increases the financial performance of companies and improves corporate sustainability in different sectors in emerging markets [67,74–76]. Table A.1 in the Appendix provides the summary of the literature review in emerging markets.

3. Methodology

In our proposed methodology, Quantum mechanics offers a new

Table 4
Average values of quantum spherical fuzzy numbers for the criteria.

	C1	C2	C3	C4	C5
C1		$\left[\sqrt{0.29}e^{j2\pi\cdot0.54}, \sqrt{0.18}e^{j2\pi\cdot0.33}, \sqrt{0.54}e^{j2\pi\cdot0.15}\right]$	$\left[\sqrt{0.28}e^{j2\pi\cdot0.52}, \sqrt{0.16}e^{j2\pi\cdot0.32}, \sqrt{0.58}e^{j2\pi\cdot0.19}\right]$	$\left[\sqrt{0.29}e^{j2\pi\cdot0.54}, \sqrt{0.18}e^{j2\pi\cdot0.33}, \sqrt{0.54}e^{j2\pi\cdot0.15}\right]$	$\left[\sqrt{0.32}e^{j2\pi\cdot0.57}, \sqrt{0.20}e^{j2\pi\cdot0.35}, \sqrt{0.48}e^{j2\pi\cdot0.09}\right]$
C2	$\left[\sqrt{0.20}e^{j2\pi\cdot0.45}, \sqrt{0.13}e^{j2\pi\cdot0.28}, \sqrt{0.67}e^{j2\pi\cdot0.27}\right]$		$\left[\sqrt{0.24}e^{j2\pi\cdot0.48}, \sqrt{0.14}e^{j2\pi\cdot0.30}, \sqrt{0.62}e^{j2\pi\cdot0.22}\right]$	$\left[\sqrt{0.33}e^{j2\pi\cdot0.57}, \sqrt{0.20}e^{j2\pi\cdot0.35}, \sqrt{0.49}e^{j2\pi\cdot0.11}\right]$	$\left[\sqrt{0.31}e^{j2\pi\cdot0.55}, \sqrt{0.19}e^{j2\pi\cdot0.34}, \sqrt{0.51}e^{j2\pi\cdot0.13}\right]$
C3	$\left[\sqrt{0.24}e^{j2\pi\cdot0.48}, \sqrt{0.14}e^{j2\pi\cdot0.30}, \sqrt{0.62}e^{j2\pi\cdot0.22}\right]$	$\left[\sqrt{0.29}e^{j2\pi\cdot0.54}, \sqrt{0.18}e^{j2\pi\cdot0.33}, \sqrt{0.54}e^{j2\pi\cdot0.15}\right]$		$\left[\sqrt{0.30}e^{j2\pi\cdot0.55}, \sqrt{0.19}e^{j2\pi\cdot0.34}, \sqrt{0.51}e^{j2\pi\cdot0.11}\right]$	$\left[\sqrt{0.27}e^{j2\pi\cdot0.51}, \sqrt{0.16}e^{j2\pi\cdot0.31}, \sqrt{0.60}e^{j2\pi\cdot0.22}\right]$
C4	$\left[\sqrt{0.25}e^{j2\pi\cdot0.50}, \sqrt{0.15}e^{j2\pi\cdot0.31}, \sqrt{0.60}e^{j2\pi\cdot0.19}\right]$	$\left[\sqrt{0.25}e^{j2\pi\cdot0.50}, \sqrt{0.15}e^{j2\pi\cdot0.31}, \sqrt{0.60}e^{j2\pi\cdot0.19}\right]$	$\left[\sqrt{0.36}e^{j2\pi\cdot0.60}, \sqrt{0.22}e^{j2\pi\cdot0.37}, \sqrt{0.42}e^{j2\pi\cdot0.03}\right]$		$\left[\sqrt{0.25}e^{j2\pi\cdot0.50}, \sqrt{0.15}e^{j2\pi\cdot0.31}, \sqrt{0.60}e^{j2\pi\cdot0.19}\right]$
C5	$\left[\sqrt{0.22}e^{j2\pi\cdot0.47}, \sqrt{0.13}e^{j2\pi\cdot0.29}, \sqrt{0.65}e^{j2\pi\cdot0.25}\right]$	$\left[\sqrt{0.27}e^{j2\pi\cdot0.52}, \sqrt{0.16}e^{j2\pi\cdot0.32}, \sqrt{0.57}e^{j2\pi\cdot0.18}\right]$	$\left[\sqrt{0.28}e^{j2\pi\cdot0.52}, \sqrt{0.16}e^{j2\pi\cdot0.32}, \sqrt{0.58}e^{j2\pi\cdot0.19}\right]$	$\left[\sqrt{0.34}e^{j2\pi\cdot0.58}, \sqrt{0.21}e^{j2\pi\cdot0.36}, \sqrt{0.45}e^{j2\pi\cdot0.07}\right]$	
C6	$\left[\sqrt{0.24}e^{j2\pi\cdot0.48}, \sqrt{0.14}e^{j2\pi\cdot0.30}, \sqrt{0.62}e^{j2\pi\cdot0.22}\right]$	$\left[\sqrt{0.28}e^{j2\pi\cdot0.52}, \sqrt{0.16}e^{j2\pi\cdot0.32}, \sqrt{0.58}e^{j2\pi\cdot0.19}\right]$	$\left[\sqrt{0.30}e^{j2\pi\cdot0.55}, \sqrt{0.19}e^{j2\pi\cdot0.34}, \sqrt{0.51}e^{j2\pi\cdot0.11}\right]$	$\left[\sqrt{0.20}e^{j2\pi\cdot0.45}, \sqrt{0.13}e^{j2\pi\cdot0.28}, \sqrt{0.67}e^{j2\pi\cdot0.27}\right]$	$\left[\sqrt{0.20}e^{j2\pi\cdot0.45}, \sqrt{0.13}e^{j2\pi\cdot0.28}, \sqrt{0.67}e^{j2\pi\cdot0.27}\right]$
C7	$\left[\sqrt{0.30}e^{j2\pi\cdot0.55}, \sqrt{0.19}e^{j2\pi\cdot0.34}, \sqrt{0.51}e^{j2\pi\cdot0.11}\right]$	$\left[\sqrt{0.30}e^{j2\pi\cdot0.54}, \sqrt{0.17}e^{j2\pi\cdot0.33}, \sqrt{0.55}e^{j2\pi\cdot0.16}\right]$	$\left[\sqrt{0.36}e^{j2\pi\cdot0.60}, \sqrt{0.22}e^{j2\pi\cdot0.37}, \sqrt{0.42}e^{j2\pi\cdot0.03}\right]$	$\left[\sqrt{0.29}e^{j2\pi\cdot0.54}, \sqrt{0.18}e^{j2\pi\cdot0.33}, \sqrt{0.54}e^{j2\pi\cdot0.15}\right]$	$\left[\sqrt{0.25}e^{j2\pi\cdot0.50}, \sqrt{0.15}e^{j2\pi\cdot0.31}, \sqrt{0.60}e^{j2\pi\cdot0.19}\right]$
C8	$\left[\sqrt{0.36}e^{j2\pi\cdot0.60}, \sqrt{0.22}e^{j2\pi\cdot0.37}, \sqrt{0.42}e^{j2\pi\cdot0.03}\right]$	$\left[\sqrt{0.24}e^{j2\pi\cdot0.48}, \sqrt{0.14}e^{j2\pi\cdot0.30}, \sqrt{0.62}e^{j2\pi\cdot0.22}\right]$	$\left[\sqrt{0.29}e^{j2\pi\cdot0.54}, \sqrt{0.18}e^{j2\pi\cdot0.33}, \sqrt{0.54}e^{j2\pi\cdot0.15}\right]$	$\left[\sqrt{0.30}e^{j2\pi\cdot0.55}, \sqrt{0.19}e^{j2\pi\cdot0.34}, \sqrt{0.51}e^{j2\pi\cdot0.11}\right]$	$\left[\sqrt{0.27}e^{j2\pi\cdot0.52}, \sqrt{0.16}e^{j2\pi\cdot0.32}, \sqrt{0.57}e^{j2\pi\cdot0.18}\right]$
C9	$\left[\sqrt{0.25}e^{j2\pi\cdot0.50}, \sqrt{0.15}e^{j2\pi\cdot0.31}, \sqrt{0.60}e^{j2\pi\cdot0.19}\right]$	$\left[\sqrt{0.30}e^{j2\pi\cdot0.54}, \sqrt{0.17}e^{j2\pi\cdot0.33}, \sqrt{0.55}e^{j2\pi\cdot0.16}\right]$	$\left[\sqrt{0.27}e^{j2\pi\cdot0.52}, \sqrt{0.16}e^{j2\pi\cdot0.32}, \sqrt{0.57}e^{j2\pi\cdot0.18}\right]$	$\left[\sqrt{0.30}e^{j2\pi\cdot0.55}, \sqrt{0.19}e^{j2\pi\cdot0.34}, \sqrt{0.51}e^{j2\pi\cdot0.11}\right]$	$\left[\sqrt{0.36}e^{j2\pi\cdot0.60}, \sqrt{0.22}e^{j2\pi\cdot0.37}, \sqrt{0.42}e^{j2\pi\cdot0.03}\right]$
C10	$\left[\sqrt{0.27}e^{j2\pi\cdot0.52}, \sqrt{0.16}e^{j2\pi\cdot0.32}, \sqrt{0.57}e^{j2\pi\cdot0.18}\right]$	$\left[\sqrt{0.36}e^{j2\pi\cdot0.60}, \sqrt{0.22}e^{j2\pi\cdot0.37}, \sqrt{0.42}e^{j2\pi\cdot0.03}\right]$	$\left[\sqrt{0.25}e^{j2\pi\cdot0.50}, \sqrt{0.15}e^{j2\pi\cdot0.31}, \sqrt{0.60}e^{j2\pi\cdot0.19}\right]$	$\left[\sqrt{0.24}e^{j2\pi\cdot0.48}, \sqrt{0.14}e^{j2\pi\cdot0.30}, \sqrt{0.62}e^{j2\pi\cdot0.22}\right]$	$\left[\sqrt{0.20}e^{j2\pi\cdot0.45}, \sqrt{0.13}e^{j2\pi\cdot0.28}, \sqrt{0.67}e^{j2\pi\cdot0.27}\right]$

	C6	C7	C8	C9	C10
C1	$\left[\sqrt{0.28}e^{j2\pi\cdot0.52}, \sqrt{0.16}e^{j2\pi\cdot0.32}, \sqrt{0.58}e^{j2\pi\cdot0.19}\right]$	$\left[\sqrt{0.36}e^{j2\pi\cdot0.60}, \sqrt{0.22}e^{j2\pi\cdot0.37}, \sqrt{0.42}e^{j2\pi\cdot0.03}\right]$	$\left[\sqrt{0.20}e^{j2\pi\cdot0.45}, \sqrt{0.13}e^{j2\pi\cdot0.28}, \sqrt{0.67}e^{j2\pi\cdot0.27}\right]$	$\left[\sqrt{0.29}e^{j2\pi\cdot0.54}, \sqrt{0.18}e^{j2\pi\cdot0.33}, \sqrt{0.54}e^{j2\pi\cdot0.15}\right]$	$\left[\sqrt{0.36}e^{j2\pi\cdot0.60}, \sqrt{0.22}e^{j2\pi\cdot0.37}, \sqrt{0.42}e^{j2\pi\cdot0.03}\right]$
C2	$\left[\sqrt{0.29}e^{j2\pi\cdot0.54}, \sqrt{0.18}e^{j2\pi\cdot0.33}, \sqrt{0.54}e^{j2\pi\cdot0.15}\right]$	$\left[\sqrt{0.22}e^{j2\pi\cdot0.47}, \sqrt{0.13}e^{j2\pi\cdot0.29}, \sqrt{0.65}e^{j2\pi\cdot0.25}\right]$	$\left[\sqrt{0.25}e^{j2\pi\cdot0.50}, \sqrt{0.15}e^{j2\pi\cdot0.31}, \sqrt{0.60}e^{j2\pi\cdot0.19}\right]$	$\left[\sqrt{0.24}e^{j2\pi\cdot0.48}, \sqrt{0.14}e^{j2\pi\cdot0.30}, \sqrt{0.62}e^{j2\pi\cdot0.22}\right]$	$\left[\sqrt{0.36}e^{j2\pi\cdot0.60}, \sqrt{0.22}e^{j2\pi\cdot0.37}, \sqrt{0.42}e^{j2\pi\cdot0.03}\right]$
C3	$\left[\sqrt{0.32}e^{j2\pi\cdot0.56}, \sqrt{0.18}e^{j2\pi\cdot0.34}, \sqrt{0.52}e^{j2\pi\cdot0.15}\right]$	$\left[\sqrt{0.29}e^{j2\pi\cdot0.54}, \sqrt{0.18}e^{j2\pi\cdot0.33}, \sqrt{0.54}e^{j2\pi\cdot0.15}\right]$	$\left[\sqrt{0.27}e^{j2\pi\cdot0.51}, \sqrt{0.15}e^{j2\pi\cdot0.31}, \sqrt{0.60}e^{j2\pi\cdot0.22}\right]$	$\left[\sqrt{0.36}e^{j2\pi\cdot0.60}, \sqrt{0.22}e^{j2\pi\cdot0.37}, \sqrt{0.42}e^{j2\pi\cdot0.03}\right]$	$\left[\sqrt{0.27}e^{j2\pi\cdot0.51}, \sqrt{0.15}e^{j2\pi\cdot0.31}, \sqrt{0.60}e^{j2\pi\cdot0.22}\right]$
C4	$\left[\sqrt{0.27}e^{j2\pi\cdot0.51}, \sqrt{0.15}e^{j2\pi\cdot0.31}, \sqrt{0.60}e^{j2\pi\cdot0.22}\right]$	$\left[\sqrt{0.36}e^{j2\pi\cdot0.60}, \sqrt{0.22}e^{j2\pi\cdot0.37}, \sqrt{0.42}e^{j2\pi\cdot0.03}\right]$	$\left[\sqrt{0.29}e^{j2\pi\cdot0.54}, \sqrt{0.18}e^{j2\pi\cdot0.33}, \sqrt{0.54}e^{j2\pi\cdot0.15}\right]$	$\left[\sqrt{0.33}e^{j2\pi\cdot0.57}, \sqrt{0.20}e^{j2\pi\cdot0.35}, \sqrt{0.49}e^{j2\pi\cdot0.11}\right]$	$\left[\sqrt{0.34}e^{j2\pi\cdot0.58}, \sqrt{0.21}e^{j2\pi\cdot0.36}, \sqrt{0.45}e^{j2\pi\cdot0.07}\right]$
C5	$\left[\sqrt{0.36}e^{j2\pi\cdot0.60}, \sqrt{0.22}e^{j2\pi\cdot0.37}, \sqrt{0.42}e^{j2\pi\cdot0.03}\right]$	$\left[\sqrt{0.36}e^{j2\pi\cdot0.60}, \sqrt{0.22}e^{j2\pi\cdot0.37}, \sqrt{0.42}e^{j2\pi\cdot0.03}\right]$	$\left[\sqrt{0.36}e^{j2\pi\cdot0.60}, \sqrt{0.22}e^{j2\pi\cdot0.37}, \sqrt{0.42}e^{j2\pi\cdot0.03}\right]$	$\left[\sqrt{0.27}e^{j2\pi\cdot0.51}, \sqrt{0.15}e^{j2\pi\cdot0.31}, \sqrt{0.60}e^{j2\pi\cdot0.22}\right]$	$\left[\sqrt{0.25}e^{j2\pi\cdot0.50}, \sqrt{0.15}e^{j2\pi\cdot0.31}, \sqrt{0.60}e^{j2\pi\cdot0.19}\right]$

(*continued on next page*)

Table 4 (*continued*)

C6		$\left[\sqrt{0.20}e^{j2\pi\cdot 0.45}, \sqrt{0.13}e^{j2\pi\cdot 0.28}, \sqrt{0.67}e^{j2\pi\cdot 0.27}\right]$	$\left[\sqrt{0.29}e^{j2\pi\cdot 0.54}, \sqrt{0.18}e^{j2\pi\cdot 0.33}, \sqrt{0.54}e^{j2\pi\cdot 0.15}\right]$	$\left[\sqrt{0.24}e^{j2\pi\cdot 0.48}, \sqrt{0.14}e^{j2\pi\cdot 0.30}, \sqrt{0.62}e^{j2\pi\cdot 0.22}\right]$	$\left[\sqrt{0.29}e^{j2\pi\cdot 0.54}, \sqrt{0.18}e^{j2\pi\cdot 0.33}, \sqrt{0.54}e^{j2\pi\cdot 0.15}\right]$
C7	$\left[\sqrt{0.30}e^{j2\pi\cdot 0.55}, \sqrt{0.19}e^{j2\pi\cdot 0.34}, \sqrt{0.51}e^{j2\pi\cdot 0.11}\right]$		$\left[\sqrt{0.22}e^{j2\pi\cdot 0.47}, \sqrt{0.13}e^{j2\pi\cdot 0.29}, \sqrt{0.65}e^{j2\pi\cdot 0.25}\right]$	$\left[\sqrt{0.36}e^{j2\pi\cdot 0.60}, \sqrt{0.22}e^{j2\pi\cdot 0.37}, \sqrt{0.42}e^{j2\pi\cdot 0.03}\right]$	$\left[\sqrt{0.27}e^{j2\pi\cdot 0.51}, \sqrt{0.15}e^{j2\pi\cdot 0.31}, \sqrt{0.60}e^{j2\pi\cdot 0.22}\right]$
C8	$\left[\sqrt{0.30}e^{j2\pi\cdot 0.55}, \sqrt{0.19}e^{j2\pi\cdot 0.34}, \sqrt{0.51}e^{j2\pi\cdot 0.11}\right]$	$\left[\sqrt{0.20}e^{j2\pi\cdot 0.45}, \sqrt{0.13}e^{j2\pi\cdot 0.28}, \sqrt{0.67}e^{j2\pi\cdot 0.27}\right]$		$\left[\sqrt{0.20}e^{j2\pi\cdot 0.45}, \sqrt{0.13}e^{j2\pi\cdot 0.28}, \sqrt{0.67}e^{j2\pi\cdot 0.27}\right]$	$\left[\sqrt{0.20}e^{j2\pi\cdot 0.45}, \sqrt{0.13}e^{j2\pi\cdot 0.28}, \sqrt{0.67}e^{j2\pi\cdot 0.27}\right]$
C9	$\left[\sqrt{0.20}e^{j2\pi\cdot 0.45}, \sqrt{0.13}e^{j2\pi\cdot 0.28}, \sqrt{0.67}e^{j2\pi\cdot 0.27}\right]$	$\left[\sqrt{0.36}e^{j2\pi\cdot 0.60}, \sqrt{0.22}e^{j2\pi\cdot 0.37}, \sqrt{0.42}e^{j2\pi\cdot 0.03}\right]$	$\left[\sqrt{0.20}e^{j2\pi\cdot 0.45}, \sqrt{0.13}e^{j2\pi\cdot 0.28}, \sqrt{0.67}e^{j2\pi\cdot 0.27}\right]$		$\left[\sqrt{0.36}e^{j2\pi\cdot 0.60}, \sqrt{0.22}e^{j2\pi\cdot 0.37}, \sqrt{0.42}e^{j2\pi\cdot 0.03}\right]$
C10	$\left[\sqrt{0.29}e^{j2\pi\cdot 0.54}, \sqrt{0.18}e^{j2\pi\cdot 0.33}, \sqrt{0.54}e^{j2\pi\cdot 0.15}\right]$	$\left[\sqrt{0.24}e^{j2\pi\cdot 0.48}, \sqrt{0.14}e^{j2\pi\cdot 0.30}, \sqrt{0.62}e^{j2\pi\cdot 0.22}\right]$	$\left[\sqrt{0.33}e^{j2\pi\cdot 0.57}, \sqrt{0.20}e^{j2\pi\cdot 0.35}, \sqrt{0.49}e^{j2\pi\cdot 0.11}\right]$	$\left[\sqrt{0.24}e^{j2\pi\cdot 0.48}, \sqrt{0.14}e^{j2\pi\cdot 0.30}, \sqrt{0.62}e^{j2\pi\cdot 0.22}\right]$	

perspective on decision-making processes by incorporating the principles of quantum theory, including amplitude and phase angle. The quantum model of mass function integrates different angles to analyze the probabilities of various conditions, enabling a more precise examination of uncertainty in complex information sets. Spherical fuzzy sets are an extension of fuzzy sets, which are widely used in decision-making. Quantum spherical fuzzy sets combine the principles of quantum mechanics and fuzzy sets, allowing for a more comprehensive analysis of uncertainty. The membership, non-membership, and hesitancy degrees of fuzzy numbers are considered together in the decision-making process within the framework of quantum spherical fuzzy sets. The golden cut, also known as the Golden Ratio, is applied in the methodology to determine the optimal ratio among the scales of spherical fuzzy sets. By utilizing the golden cut, the amplitude of non-membership and hesitancy degrees can be defined, enhancing the accuracy of decision-making under uncertainty.

The advantages of the proposed model can be listed as by improved the accuracy, enhanced understanding of uncertainty, wide applicability, integration of multiple parameters, theoretical grounding. Accordingly, the utilization of quantum spherical fuzzy sets with the golden cut enables a more accurate representation of complex decision-making problems. By incorporating the amplitude and phase angles, the methodology captures nuanced degrees of membership, non-membership, and hesitancy, leading to more precise results. The methodology also provides a comprehensive characterization and analysis of uncertainty in decision-making. The phase angle plays a crucial role in understanding the relationships and relative importance of different components within the quantum spherical fuzzy sets. It facilitates a deeper exploration of uncertainty, leading to a more informed decision-making process. Again, the concept of phase angles has widespread application in various disciplines such as physics, engineering, and mathematics. By incorporating phase angles within the framework of quantum spherical fuzzy sets, the methodology extends its applicability to diverse domains, allowing for the analysis and synchronization of signals, waveforms, and oscillations. Another point of view is about the methodology that considers the simultaneous evaluation of membership, non-membership, and hesitancy degrees in decision-making. This comprehensive approach provides a holistic understanding of uncertain information, enabling decision-makers to make informed choices based on multiple parameters simultaneously. So, the methodology is built upon established theoretical foundations, including quantum mechanics, fuzzy sets, and the golden cut. By integrating these theories, the methodology benefits from the robustness and rigor of existing academic frameworks, enhancing its credibility and reliability. The details of the proposed methodology are given by the following sections.

3.1. Quantum spherical fuzzy sets with golden cut

Quantum mechanics provides a new outlook to the decision-making process in bringing solutions to real-world problems [77,78]. The probability approach could be considered more efficiently by using the quantum theory with the amplitude and the phase angle items. Hence, the quantum model of mass function includes different angles to understand the probabilities of several conditions [23,24]. Thus, the uncertainty could be studied more accurately in the complex information set of quantum logic. The probability of quantum mass function with the amplitude and the phase angle is presented in Eq. (1) to 3 [79,80]:

$$Q(|u\rangle) = \varphi e^{j\theta} \tag{1}$$

$$|\varsigma\rangle = \{|u_1\rangle, |u_2\rangle, \ldots, |u_n\rangle\} \tag{2}$$

$$\sum_{|u\rangle \subseteq |\varsigma\rangle} |Q(|u\rangle)| = 1 \tag{3}$$

where ς is the set of collective exhaustive events $|u_i\rangle$, $|Q(|u\rangle)| = \varphi^2$ is the

Table 5
Score function of the criteria for quantum spherical fuzzy sets.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
C1	0.000	1.266	1.284	1.243	1.245	1.284	1.236	1.236	1.266	1.236
C2	1.236	0.000	1.263	1.269	1.259	1.243	1.243	1.236	1.263	1.236
C3	1.243	1.266	0.000	1.236	1.297	1.300	1.243	1.297	1.236	1.297
C4	1.236	1.236	1.236	0.000	1.236	1.297	1.236	1.266	1.269	1.247
C5	1.243	1.263	1.284	1.247	0.000	1.236	1.236	1.236	1.297	1.236
C6	1.243	1.284	1.236	1.236	1.236	0.000	1.236	1.243	1.263	1.266
C7	1.236	1.285	1.236	1.266	1.236	1.236	0.000	1.243	1.236	1.243
C8	1.236	1.243	1.243	1.236	1.263	1.236	1.236	0.000	1.236	1.236
C9	1.236	1.285	1.297	1.236	1.236	1.236	1.236	1.236	0.000	1.236
C10	1.263	1.236	1.236	1.263	1.236	1.243	1.263	1.269	1.263	0.000

Table 6
Normalized relation matrix.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
C1	0.000	0.111	0.113	0.109	0.109	0.113	0.108	0.108	0.111	0.108
C2	0.108	0.000	0.111	0.111	0.110	0.109	0.109	0.108	0.111	0.108
C3	0.109	0.111	0.000	0.108	0.114	0.114	0.109	0.114	0.108	0.114
C4	0.108	0.108	0.108	0.000	0.108	0.114	0.108	0.111	0.111	0.109
C5	0.109	0.111	0.113	0.109	0.000	0.108	0.108	0.108	0.114	0.108
C6	0.109	0.113	0.108	0.108	0.108	0.000	0.108	0.109	0.111	0.111
C7	0.108	0.113	0.108	0.111	0.108	0.108	0.000	0.109	0.108	0.109
C8	0.108	0.109	0.109	0.108	0.111	0.108	0.108	0.000	0.108	0.108
C9	0.108	0.113	0.114	0.108	0.108	0.108	0.108	0.108	0.000	0.108
C10	0.111	0.108	0.108	0.111	0.108	0.109	0.111	0.111	0.111	0.000

Table 7
Total relation matrix and the impact directions.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	Impact directions
C1	7.289	7.501	7.476	7.423	7.431	7.473	7.383	7.441	7.482	7.423	C1→(C2, C3, C4, C5, C6, C8, C9, C10)
C2	7.359	7.373	7.446	7.397	7.404	7.442	7.356	7.412	7.453	7.395	C2→(C3, C6, C8, C9)
C3	7.456	7.571	7.444	7.492	7.504	7.544	7.452	7.514	7.549	7.497	C3→(C1, C2, C4, C5, C6, C7, C8, C9, C10)
C4	7.365	7.477	7.450	7.303	7.408	7.452	7.361	7.420	7.459	7.402	C4→(C2, C3, C6, C8, C9)
C5	7.377	7.490	7.465	7.413	7.322	7.459	7.373	7.430	7.473	7.413	C5→(C2, C3, C4, C6, C8, C9, C10)
C6	7.356	7.471	7.440	7.391	7.399	7.340	7.352	7.409	7.449	7.394	C6→(C2, C3, C9)
C7	7.340	7.455	7.425	7.378	7.383	7.423	7.239	7.394	7.432	7.377	C7→(C2, C3, C6, C9)
C8	7.310	7.422	7.395	7.346	7.355	7.392	7.307	7.266	7.402	7.346	C8→(C2)
C9	7.351	7.466	7.440	7.387	7.394	7.433	7.347	7.404	7.345	7.387	C9→(C2, C3, C6)
C10	7.374	7.484	7.457	7.410	7.416	7.456	7.371	7.428	7.467	7.311	C10→(C2, C3, C5, C6, C8, C9)

Table 8
Influence degrees and weights of the criteria.

	D	E	D + E	D−E	Weighting results	Weighting priorities
C1	74.323	73.576	147.899	0.747	0.0998	8
C2	74.039	74.711	148.750	−0.672	0.1003	2
C3	75.020	74.436	149.457	0.584	0.1008	1
C4	74.098	73.940	148.038	0.158	0.0999	7
C5	74.214	74.016	148.230	0.197	0.1000	5
C6	74.001	74.415	148.416	−0.413	0.1001	4
C7	73.845	73.542	147.387	0.303	0.0994	10
C8	73.541	74.118	147.660	−0.577	0.0996	9
C9	73.955	74.511	148.466	−0.556	0.1002	3
C10	74.174	73.945	148.120	0.229	0.0999	6

amplitude result for the probability of an event $|u\rangle$ in the form of quantum logic. $0 \leq \varphi^2 \leq 1$ and θ^2 is the phase angle of the event $|u\rangle$. $|\varphi_1|^2$ is the belief degree to $|u\rangle$, and the value of θ is the phase angle of $|u\rangle$ with the range $[0, 360^0]$.

Decision-making problems generally include several qualitative and indefinite evaluations that cannot be defined with exact values. This vagueness raises the extensions of decision-making approaches for providing more accurate results under uncertainty. The fuzzy sets introduced by [81] are one of the most prominent methods for solving

Table 9
Selected sector alternatives of emerging economies for the ESG performance.

Alternatives	Description
Financial Services (A1)	Banks, Consumer Finance, Capital Markets, Mortgage Real Estate Investment Trusts (REITS), Insurance
Consumer Defensive (A2)	Essentials for everyday use
Technology (A3)	Software & Services, Technology Hardware and Equipment, Electronic Equipment's, Storage, Chipsets, Semiconductors and Equipment
Real Estate(A4)	Real Estates and Services
Energy (A5)	Energy Equipment & Services, Oil, Gas and Consumable Fuels, Alternative Energy, Renewable energy
Communication Services(A6)	Telecommunication Services, Media, and Entertainment.
Healthcare (A7)	Health Care Equipment, Services, Pharmaceuticals, Biotechnology, Life Sciences Tools, and Services
Consumer Cyclical (A8)	Automotive, Housing, Entertainment, and Retail
Basic Materials (A9)	Chemicals, Constructions Materials, Containers, Metals, mining, Paper, and Forest Products
Utilities (A10)	Electric Utilities, Gas Utilities, Water Utilities

Table 10
Linguistic evaluations of decision makers for the alternatives.

Decision maker 1										
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	P	F	B	P	P	P	P	P	G	P
A2	G	G	P	B	G	G	G	P	B	P
A3	F	P	P	P	P	P	G	P	B	G
A4	F	F	B	P	P	G	B	P	P	B
A5	P	G	P	B	B	F	P	B	P	F
A6	F	F	P	P	P	G	G	G	P	F
A7	P	P	B	F	P	G	G	G	G	P
A8	B	P	G	G	G	G	P	G	G	P
A9	P	P	P	G	B	P	B	P	G	G
A10	P	P	F	P	P	G	P	B	B	P
Decision maker 2										
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	P	G	B	B	B	P	P	P	G	B
A2	G	G	P	B	B	G	G	P	B	P
A3	G	P	P	P	P	P	G	P	B	G
A4	B	B	B	P	P	G	B	P	P	B
A5	P	G	P	B	B	F	B	B	P	F
A6	F	B	B	P	P	G	P	G	P	B
A7	P	P	B	F	P	G	G	G	G	P
A8	B	B	B	G	G	G	P	G	G	P
A9	G	P	P	B	B	P	G	G	G	B
A10	G	B	B	P	P	G	G	B	B	P
Decision maker 3										
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	P	B	B	F	P	P	P	G	G	B
A2	B	G	F	B	P	B	G	G	B	P
A3	F	B	B	P	P	P	B	P	P	B
A4	F	F	B	P	P	G	B	P	P	B
A5	P	P	B	B	G	B	P	G	P	B
A6	F	F	B	P	B	G	G	G	P	F
A7	P	P	P	F	P	G	G	G	B	B
A8	B	P	B	F	B	G	P	G	B	P
A9	P	P	G	B	B	P	B	P	F	B
A10	P	B	B	P	P	G	P	B	B	P

P: Poor; G: Good; F: Fair; B: Best.

complex problems of decision-making. After the introduction of fuzzy logic, some extensions have been presented such as type 2 and intuitionistic fuzzy sets [82,83]. Recently, Spherical fuzzy sets are applied to increase the precise results with the generalized form of Neutrosophic and Pythagorean fuzzy numbers. The phase angle represents a pivotal parameter within the realm of quantum spherical fuzzy sets. Its application contributes to the comprehensive characterization and analysis of complex information sets, thereby enabling a more accurate study of uncertainty and decision-making problems. In this method, the membership, non-membership, and hesitancy degrees of fuzzy numbers are considered together in the decision-making process with Quantum mechanics. The limitation of this extension is that the square sum of the membership μ, non-membership ν, and hesitation π parameters are between 0 and 1. The definition of Spherical fuzzy sets $\widetilde{A}_S$ is given by Eqs. (4) and (5) [84].

$$\widetilde{A}_S = \left\{ \left\langle u, \left(\mu_{\tilde{A}_S}(u), v_{\tilde{A}_S}(u), h_{\tilde{A}_S}(u) \right) \middle| u \in U \right\rangle \right\} \tag{4}$$

$$0 \le \mu^2_{\tilde{A}_S}(u) + v^2_{\tilde{A}_S}(u) + h^2_{\tilde{A}_S}(u) \le 1, \forall_u \in U \tag{5}$$

The probability of quantum theory could be generalized in the form of Spherical fuzzy sets to solve the complex decision-making problems with the amplitude and the phase angles of the comprehensive set as indicated in Eq. (6) [25,26,85].

$$\left|\varsigma_{\tilde{A}_S}\right\rangle = \left\{ \left\langle u, \left(\varsigma_{\mu_{\tilde{A}_S}}(u), \varsigma_{v_{\tilde{A}_S}}(u), \varsigma_{h_{\tilde{A}_S}}(u) \right) \middle| u \in 2^{\left|\varsigma_{\tilde{A}_S}\right\rangle} \right\rangle \right\} \tag{6}$$

where, $\varsigma_{\mu_{\tilde{A}_S}}$, $\varsigma_{v_{\tilde{A}_S}}$, and $\varsigma_{h_{\tilde{A}_S}}$ are the membership, and non-membership hesitant degrees of Quantum Spherical fuzzy sets, respectively.

However, the Quantum Spherical fuzzy numbers ς are formulized with the amplitude and phase angles of the fuzzy sets as follows:

$$\varsigma = \left[\varsigma_\mu . e^{j2\pi.\alpha}, \varsigma_v . e^{j2\pi.\gamma}, \varsigma_h . e^{j2\pi.\beta} \right] \tag{7}$$

$$\varphi^2 = \left| \varsigma_\mu(|u_i\rangle) \right| \tag{8}$$

where, ς_μ, ς_v, and ς_h are the amplitudes of quantum membership, non-membership, and hesitancy degrees as α, γ, and β are the set of θ phase angles respectively. φ^2 defines the amplitude of the membership function ς_μ of quantum fuzzy sets.

Defining the right membership, non-membership, and hesitancy degrees in the Spherical fuzzy sets is still an outstanding problem of decision-making methods and there is no consensus in the determination of the membership and other scales. To answer this concern, the golden cut could be used to construct the optimal ratio among the scales of Spherical fuzzy sets, also known as the Golden Ratio. It sheds light on the specific patterns of geometry problems. Golden Ratio was first studied by Greek mathematicians to discover the relationship between geometric figures. The forwarded studies have been illustrated by associating the Fibonacci numbers with the golden ratio [86,87].

The golden cut can be defined with the division of extreme and mean ratio in a straight line including the large and small quantities as in Eq. (9):

$$G = \frac{x}{b} \tag{9}$$

Where $x > b > 0$ and G is golden cut, x defines the large quantity and b is the small quantity of the straight line.

The algebraic form of golden cut can also be given as in Eq. (10):

$$G = \frac{1 + \sqrt{5}}{2} = 1.618\ldots \tag{10}$$

The amplitude of non-membership degrees for the quantum spherical fuzzy sets is defined with *"golden cut"* by Eq. (11):

$$\varsigma_v = \frac{\varsigma_\mu}{G} \tag{11}$$

The amplitude of hesitancy degrees is presented in Eq. (12):

$$\varsigma_h = 1 - \varsigma_\mu - \varsigma_v \tag{12}$$

Accordingly, the phase angle of the quantum spherical fuzzy sets is given as follows:

$$\alpha = \left| \varsigma_\mu(|u_i\rangle) \right| \tag{13}$$

α is the phase angle of the membership degrees for the probability of an event $|u\rangle$ in the form of quantum spherical fuzzy sets.

The phase angle of non-member degrees γ is determined by Eq. (14):

$$\gamma = \frac{\alpha}{G} \tag{14}$$

The phase angle of hesitancy degrees β is constructed by Eq. (15):

Table 11
Average values of quantum spherical fuzzy numbers for the alternatives.

	C1	C2	C3	C4	C5
A1	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.31}e^{j2\pi.0.55},\\ \sqrt{0.19}e^{j2\pi.0.34},\\ \sqrt{0.51}e^{j2\pi.0.13}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.36}e^{j2\pi.0.60},\\ \sqrt{0.22}e^{j2\pi.0.37},\\ \sqrt{0.42}e^{j2\pi.0.03}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.28}e^{j2\pi.0.52},\\ \sqrt{0.16}e^{j2\pi.0.32},\\ \sqrt{0.58}e^{j2\pi.0.19}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.27}e^{j2\pi.0.52},\\ \sqrt{0.16}e^{j2\pi.0.32},\\ \sqrt{0.57}e^{j2\pi.0.18}\end{bmatrix}$
A2	$\begin{bmatrix}\sqrt{0.32}e^{j2\pi.0.57},\\ \sqrt{0.20}e^{j2\pi.0.35},\\ \sqrt{0.48}e^{j2\pi.0.09}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.30}e^{j2\pi.0.55},\\ \sqrt{0.19}e^{j2\pi.0.34},\\ \sqrt{0.51}e^{j2\pi.0.11}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.22}e^{j2\pi.0.47},\\ \sqrt{0.13}e^{j2\pi.0.29},\\ \sqrt{0.65}e^{j2\pi.0.25}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.36}e^{j2\pi.0.60},\\ \sqrt{0.22}e^{j2\pi.0.37},\\ \sqrt{0.42}e^{j2\pi.0.03}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.30}e^{j2\pi.0.55},\\ \sqrt{0.19}e^{j2\pi.0.34},\\ \sqrt{0.51}e^{j2\pi.0.11}\end{bmatrix}$
A3	$\begin{bmatrix}\sqrt{0.27}e^{j2\pi.0.52},\\ \sqrt{0.16}e^{j2\pi.0.32},\\ \sqrt{0.57}e^{j2\pi.0.18}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.27}e^{j2\pi.0.52},\\ \sqrt{0.16}e^{j2\pi.0.32},\\ \sqrt{0.57}e^{j2\pi.0.18}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.27}e^{j2\pi.0.52},\\ \sqrt{0.16}e^{j2\pi.0.32},\\ \sqrt{0.57}e^{j2\pi.0.18}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$
A4	$\begin{bmatrix}\sqrt{0.29}e^{j2\pi.0.54},\\ \sqrt{0.18}e^{j2\pi.0.33},\\ \sqrt{0.54}e^{j2\pi.0.15}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.29}e^{j2\pi.0.54},\\ \sqrt{0.18}e^{j2\pi.0.33},\\ \sqrt{0.54}e^{j2\pi.0.15}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.36}e^{j2\pi.0.60},\\ \sqrt{0.22}e^{j2\pi.0.37},\\ \sqrt{0.42}e^{j2\pi.0.03}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$
A5	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.27}e^{j2\pi.0.52},\\ \sqrt{0.16}e^{j2\pi.0.32},\\ \sqrt{0.57}e^{j2\pi.0.18}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.27}e^{j2\pi.0.52},\\ \sqrt{0.16}e^{j2\pi.0.32},\\ \sqrt{0.57}e^{j2\pi.0.18}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.36}e^{j2\pi.0.60},\\ \sqrt{0.22}e^{j2\pi.0.37},\\ \sqrt{0.42}e^{j2\pi.0.03}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.34}e^{j2\pi.0.58},\\ \sqrt{0.21}e^{j2\pi.0.36},\\ \sqrt{0.45}e^{j2\pi.0.07}\end{bmatrix}$
A6	$\begin{bmatrix}\sqrt{0.25}e^{j2\pi.0.50},\\ \sqrt{0.15}e^{j2\pi.0.31},\\ \sqrt{0.60}e^{j2\pi.0.19}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.29}e^{j2\pi.0.54},\\ \sqrt{0.18}e^{j2\pi.0.33},\\ \sqrt{0.54}e^{j2\pi.0.15}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.32}e^{j2\pi.0.57},\\ \sqrt{0.20}e^{j2\pi.0.35},\\ \sqrt{0.48}e^{j2\pi.0.09}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.27}e^{j2\pi.0.52},\\ \sqrt{0.16}e^{j2\pi.0.32},\\ \sqrt{0.57}e^{j2\pi.0.18}\end{bmatrix}$
A7	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.32}e^{j2\pi.0.57},\\ \sqrt{0.20}e^{j2\pi.0.35},\\ \sqrt{0.48}e^{j2\pi.0.09}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.25}e^{j2\pi.0.50},\\ \sqrt{0.15}e^{j2\pi.0.31},\\ \sqrt{0.60}e^{j2\pi.0.19}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.27}e^{j2\pi.0.52},\\ \sqrt{0.16}e^{j2\pi.0.32},\\ \sqrt{0.57}e^{j2\pi.0.18}\end{bmatrix}$
A8	$\begin{bmatrix}\sqrt{0.36}e^{j2\pi.0.60},\\ \sqrt{0.22}e^{j2\pi.0.37},\\ \sqrt{0.42}e^{j2\pi.0.03}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.27}e^{j2\pi.0.52},\\ \sqrt{0.16}e^{j2\pi.0.32},\\ \sqrt{0.57}e^{j2\pi.0.18}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.34}e^{j2\pi.0.58},\\ \sqrt{0.21}e^{j2\pi.0.36},\\ \sqrt{0.45}e^{j2\pi.0.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.29}e^{j2\pi.0.54},\\ \sqrt{0.18}e^{j2\pi.0.33},\\ \sqrt{0.54}e^{j2\pi.0.15}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.32}e^{j2\pi.0.57},\\ \sqrt{0.20}e^{j2\pi.0.35},\\ \sqrt{0.48}e^{j2\pi.0.09}\end{bmatrix}$
A9	$\begin{bmatrix}\sqrt{0.24}e^{j2\pi.0.48},\\ \sqrt{0.14}e^{j2\pi.0.30},\\ \sqrt{0.62}e^{j2\pi.0.22}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.24}e^{j2\pi.0.48},\\ \sqrt{0.14}e^{j2\pi.0.30},\\ \sqrt{0.62}e^{j2\pi.0.22}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.34}e^{j2\pi.0.58},\\ \sqrt{0.21}e^{j2\pi.0.36},\\ \sqrt{0.45}e^{j2\pi.0.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.36}e^{j2\pi.0.60},\\ \sqrt{0.22}e^{j2\pi.0.37},\\ \sqrt{0.42}e^{j2\pi.0.03}\end{bmatrix}$
A10	$\begin{bmatrix}\sqrt{0.24}e^{j2\pi.0.48},\\ \sqrt{0.14}e^{j2\pi.0.30},\\ \sqrt{0.62}e^{j2\pi.0.22}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.32}e^{j2\pi.0.57},\\ \sqrt{0.20}e^{j2\pi.0.35},\\ \sqrt{0.48}e^{j2\pi.0.09}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.33}e^{j2\pi.0.57},\\ \sqrt{0.20}e^{j2\pi.0.35},\\ \sqrt{0.49}e^{j2\pi.0.11}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$

	C6	C7	C8	C9	C10
A1	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.24}e^{j2\pi.0.48},\\ \sqrt{0.14}e^{j2\pi.0.30},\\ \sqrt{0.62}e^{j2\pi.0.22}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.30}e^{j2\pi.0.55},\\ \sqrt{0.19}e^{j2\pi.0.34},\\ \sqrt{0.51}e^{j2\pi.0.11}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.32}e^{j2\pi.0.57},\\ \sqrt{0.20}e^{j2\pi.0.35},\\ \sqrt{0.48}e^{j2\pi.0.09}\end{bmatrix}$
A2	$\begin{bmatrix}\sqrt{0.32}e^{j2\pi.0.57},\\ \sqrt{0.20}e^{j2\pi.0.35},\\ \sqrt{0.48}e^{j2\pi.0.09}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.30}e^{j2\pi.0.55},\\ \sqrt{0.19}e^{j2\pi.0.34},\\ \sqrt{0.51}e^{j2\pi.0.11}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.24}e^{j2\pi.0.48},\\ \sqrt{0.14}e^{j2\pi.0.30},\\ \sqrt{0.62}e^{j2\pi.0.22}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.36}e^{j2\pi.0.60},\\ \sqrt{0.22}e^{j2\pi.0.37},\\ \sqrt{0.42}e^{j2\pi.0.03}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$
A3	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.32}e^{j2\pi.0.57},\\ \sqrt{0.20}e^{j2\pi.0.35},\\ \sqrt{0.48}e^{j2\pi.0.09}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.32}e^{j2\pi.0.57},\\ \sqrt{0.20}e^{j2\pi.0.35},\\ \sqrt{0.48}e^{j2\pi.0.09}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.32}e^{j2\pi.0.57},\\ \sqrt{0.20}e^{j2\pi.0.35},\\ \sqrt{0.48}e^{j2\pi.0.09}\end{bmatrix}$
A4	$\begin{bmatrix}\sqrt{0.30}e^{j2\pi.0.55},\\ \sqrt{0.19}e^{j2\pi.0.34},\\ \sqrt{0.51}e^{j2\pi.0.11}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.36}e^{j2\pi.0.60},\\ \sqrt{0.22}e^{j2\pi.0.37},\\ \sqrt{0.42}e^{j2\pi.0.03}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.36}e^{j2\pi.0.60},\\ \sqrt{0.22}e^{j2\pi.0.37},\\ \sqrt{0.42}e^{j2\pi.0.03}\end{bmatrix}$
A5	$\begin{bmatrix}\sqrt{0.29}e^{j2\pi.0.54},\\ \sqrt{0.18}e^{j2\pi.0.33},\\ \sqrt{0.54}e^{j2\pi.0.15}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.27}e^{j2\pi.0.52},\\ \sqrt{0.16}e^{j2\pi.0.32},\\ \sqrt{0.57}e^{j2\pi.0.18}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.34}e^{j2\pi.0.58},\\ \sqrt{0.21}e^{j2\pi.0.36},\\ \sqrt{0.45}e^{j2\pi.0.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.20}e^{j2\pi.0.45},\\ \sqrt{0.13}e^{j2\pi.0.28},\\ \sqrt{0.67}e^{j2\pi.0.27}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.29}e^{j2\pi.0.54},\\ \sqrt{0.18}e^{j2\pi.0.33},\\ \sqrt{0.54}e^{j2\pi.0.15}\end{bmatrix}$

(*continued on next page*)

Table 11 (*continued*)

	C1	C2	C3	C4	C5
A6	$\begin{bmatrix}\sqrt{0.30}\,e^{j2\pi.0.55},\\ \sqrt{0.19}\,e^{j2\pi.0.34},\\ \sqrt{0.51}\,e^{j2\pi.0.11}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.27}\,e^{j2\pi.0.52},\\ \sqrt{0.16}\,e^{j2\pi.0.32},\\ \sqrt{0.57}\,e^{j2\pi.0.18}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.30}\,e^{j2\pi.0.55},\\ \sqrt{0.19}\,e^{j2\pi.0.34},\\ \sqrt{0.51}\,e^{j2\pi.0.11}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.20}\,e^{j2\pi.0.45},\\ \sqrt{0.13}\,e^{j2\pi.0.28},\\ \sqrt{0.67}\,e^{j2\pi.0.27}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.29}\,e^{j2\pi.0.54},\\ \sqrt{0.18}\,e^{j2\pi.0.33},\\ \sqrt{0.54}\,e^{j2\pi.0.15}\end{bmatrix}$
A7	$\begin{bmatrix}\sqrt{0.30}\,e^{j2\pi.0.55},\\ \sqrt{0.19}\,e^{j2\pi.0.34},\\ \sqrt{0.51}\,e^{j2\pi.0.11}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.30}\,e^{j2\pi.0.55},\\ \sqrt{0.19}\,e^{j2\pi.0.34},\\ \sqrt{0.51}\,e^{j2\pi.0.11}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.30}\,e^{j2\pi.0.55},\\ \sqrt{0.19}\,e^{j2\pi.0.34},\\ \sqrt{0.51}\,e^{j2\pi.0.11}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.32}\,e^{j2\pi.0.57},\\ \sqrt{0.20}\,e^{j2\pi.0.35},\\ \sqrt{0.48}\,e^{j2\pi.0.09}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.27}\,e^{j2\pi.0.51},\\ \sqrt{0.15}\,e^{j2\pi.0.31},\\ \sqrt{0.60}\,e^{j2\pi.0.22}\end{bmatrix}$
A8	$\begin{bmatrix}\sqrt{0.30}\,e^{j2\pi.0.55},\\ \sqrt{0.19}\,e^{j2\pi.0.34},\\ \sqrt{0.51}\,e^{j2\pi.0.11}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.20}\,e^{j2\pi.0.45},\\ \sqrt{0.13}\,e^{j2\pi.0.28},\\ \sqrt{0.67}\,e^{j2\pi.0.27}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.30}\,e^{j2\pi.0.55},\\ \sqrt{0.19}\,e^{j2\pi.0.34},\\ \sqrt{0.51}\,e^{j2\pi.0.11}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.32}\,e^{j2\pi.0.57},\\ \sqrt{0.20}\,e^{j2\pi.0.35},\\ \sqrt{0.48}\,e^{j2\pi.0.09}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.20}\,e^{j2\pi.0.45},\\ \sqrt{0.13}\,e^{j2\pi.0.28},\\ \sqrt{0.67}\,e^{j2\pi.0.27}\end{bmatrix}$
A9	$\begin{bmatrix}\sqrt{0.20}\,e^{j2\pi.0.45},\\ \sqrt{0.13}\,e^{j2\pi.0.28},\\ \sqrt{0.67}\,e^{j2\pi.0.27}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.34}\,e^{j2\pi.0.58},\\ \sqrt{0.21}\,e^{j2\pi.0.36},\\ \sqrt{0.45}\,e^{j2\pi.0.07}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.24}\,e^{j2\pi.0.48},\\ \sqrt{0.14}\,e^{j2\pi.0.30},\\ \sqrt{0.62}\,e^{j2\pi.0.22}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.29}\,e^{j2\pi.0.54},\\ \sqrt{0.18}\,e^{j2\pi.0.33},\\ \sqrt{0.54}\,e^{j2\pi.0.15}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.34}\,e^{j2\pi.0.58},\\ \sqrt{0.21}\,e^{j2\pi.0.36},\\ \sqrt{0.45}\,e^{j2\pi.0.07}\end{bmatrix}$
A10	$\begin{bmatrix}\sqrt{0.30}\,e^{j2\pi.0.55},\\ \sqrt{0.19}\,e^{j2\pi.0.34},\\ \sqrt{0.51}\,e^{j2\pi.0.11}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.24}\,e^{j2\pi.0.48},\\ \sqrt{0.14}\,e^{j2\pi.0.30},\\ \sqrt{0.62}\,e^{j2\pi.0.22}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.36}\,e^{j2\pi.0.60},\\ \sqrt{0.22}\,e^{j2\pi.0.37},\\ \sqrt{0.42}\,e^{j2\pi.0.03}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.36}\,e^{j2\pi.0.60},\\ \sqrt{0.22}\,e^{j2\pi.0.37},\\ \sqrt{0.42}\,e^{j2\pi.0.03}\end{bmatrix}$	$\begin{bmatrix}\sqrt{0.20}\,e^{j2\pi.0.45},\\ \sqrt{0.13}\,e^{j2\pi.0.28},\\ \sqrt{0.67}\,e^{j2\pi.0.27}\end{bmatrix}$

$$\beta = 1 - \alpha - \gamma \tag{15}$$

The phase angle, a foundational concept with far-reaching implications in various academic disciplines, embodies profound significance in characterizing complex systems and elucidating relationships between diverse components or signals. This pivotal notion finds wide application in diverse fields such as physics, engineering, mathematics, and signal processing, where its role in understanding oscillatory phenomena and the timing of events is paramount. Measured in radians or degrees, the phase angle represents a fraction of a complete cycle or period. A phase angle of 0 degrees signifies perfect alignment or synchronization, while a phase angle of 180 degrees denotes complete opposition or anti-alignment.

In the realm of quantum mechanics, the phase angle assumes a preeminent position, unveiling novel insights into the decision-making process and yielding innovative solutions to real-world predicaments. Leveraging the amplitude and phase angle items within quantum theory, the probability approach acquires heightened efficiency, empowering a more profound grasp of probabilities associated with a multitude of conditions. Consequently, the quantum model of mass function employs diverse angles, offering a comprehensive means to decipher intricate probabilities in the context of complex information sets governed by quantum logic, enabling more precise studies of uncertainty.

Furthermore, quantum spherical fuzzy sets, which have emerged as a prominent approach in addressing complex decision-making challenges, incorporate the phase angle as a pivotal parameter. This profound characteristic enriches the comprehensive characterization and analysis of intricate information sets, elevating the exploration of uncertainty and decision-making problems to new heights. Moreover, by integrating the membership, non-membership, and hesitancy degrees of fuzzy numbers with quantum mechanics, researchers can foster a more incisive understanding and resolution of complex decision-making quandaries.

The formulation of quantum spherical fuzzy sets introduces the phase angle as an essential component in the representation of the quantum fuzzy numbers ς. This novel formulation comprises three vital components: $\varsigma_\mu.e^{j2\pi.\alpha}, \varsigma_\nu.e^{j2\pi.\gamma}, \varsigma_h.e^{j2\pi.\beta}$, with α, γ, and β embodying sets of θ phase angles, respectively. Through this unique representation, the phase angles encode invaluable insights into the relative positioning and phase relationships of the corresponding degrees within the intricate complex plane. Intriguingly, the phase angle α, an indispensable indicator of the membership degrees $|u_i\rangle$, profoundly influences the probability representation of quantum spherical fuzzy sets (as stated in Eq. (13)). By meticulously analyzing the phase angle α, researchers can gain profound insights into the phase relationships between a myriad of events or conditions encapsulated by quantum spherical fuzzy sets, thereby enriching the understanding of uncertainty and decision-making processes in this domain.

Despite the complexity and importance of phase angles in decision-making, the challenge of defining precise membership, non-membership, and hesitancy degrees within spherical fuzzy sets endures as an outstanding academic pursuit. The lack of a consensus on the determination of these parameters necessitates novel approaches to resolving this critical issue. To this end, the proposition of the golden cut, an iconic mathematical concept attributed to ancient Greek mathematicians, emerges as a potential remedy. The quantum mechanics, with the golden ratio, offers profound insights into the optimal ratios among the scales of spherical fuzzy sets, illuminating specific geometric patterns in complex decision-making problems.

X_1 and X_2 are two universes and $\widetilde{A}_\varsigma = \left(\varsigma_{\mu_{\widetilde{A}}} e^{j2\pi.\alpha_{\widetilde{A}}}, \varsigma_{\nu_{\widetilde{A}}} e^{j2\pi.\gamma_{\widetilde{A}}}, \varsigma_{h_{\widetilde{A}}} e^{j2\pi.\beta_{\widetilde{A}}}\right)$ and $\widetilde{B}_\varsigma = \left(\varsigma_{\mu_{\widetilde{B}}}.e^{j2\pi.\alpha_{\widetilde{B}}}, \varsigma_{\nu_{\widetilde{B}}}.e^{j2\pi.\gamma_{\widetilde{B}}}, \varsigma_{h_{\widetilde{B}}}.e^{j2\pi.\beta_{\widetilde{B}}}\right)$ are two quantum spherical fuzzy sets from the universe of discourse X_1 and X_2. The operations of quantum spherical fuzzy numbers are shown in the following equations [88,89]:

$$\lambda^{*}\widetilde{A}_{\varsigma}=\Bigg\{\left(1-\left(1-\varsigma_{\mu_{\tilde{A}}}{}^{2}\right)^{\lambda}\right)^{\frac{1}{2}}e^{j2\pi.\left(1-\left(1-\left(\frac{\alpha_{\tilde{A}}}{2\pi}\right)^{2}\right)^{\lambda}\right)^{\frac{1}{2}}},\varsigma_{\nu_{\tilde{A}}}{}^{\lambda}\times e^{j2\pi.\left(\frac{\gamma_{\tilde{A}}}{2\pi}\right)^{\lambda}},\left(\left(1-\varsigma_{h_{\tilde{A}}}{}^{2}\right)^{\lambda}-\left(1-\varsigma_{\mu_{\tilde{A}}}{}^{2}-\varsigma_{h_{\tilde{A}}}{}^{2}\right)^{\lambda}\right)^{\frac{1}{2}}\times e^{j2\pi.\left(\left(1-\left(\frac{\beta_{\tilde{A}}}{2\pi}\right)^{2}\right)^{\lambda}-\left(1-\left(\frac{\alpha_{\tilde{A}}}{2\pi}\right)^{2}-\left(\frac{\beta_{\tilde{A}}}{2\pi}\right)^{2}\right)^{\lambda}\right)^{\frac{1}{2}}}\Bigg\},\lambda>0 \tag{16}$$

$$\widetilde{A}_{\varsigma}{}^{\lambda}=\Bigg\{\varsigma_{\mu_{\tilde{A}}}{}^{\lambda}e^{j2\pi.\left(\frac{\alpha_{\tilde{A}}}{2\pi}\right)^{\lambda}},\left(1-\left(1-\varsigma_{\nu_{\tilde{A}}}{}^{2}\right)^{\lambda}\right)^{\frac{1}{2}}\times e^{j2\pi.\left(1-\left(1-\left(\frac{\gamma_{\tilde{A}}}{2\pi}\right)^{2}\right)^{\lambda}\right)^{\frac{1}{2}}},\left(\left(1-\varsigma_{\nu_{\tilde{A}}}{}^{2}\right)^{\lambda}-\left(1-\varsigma_{\nu_{\tilde{A}}}{}^{2}-\varsigma_{h_{\tilde{A}}}{}^{2}\right)^{\lambda}\right)^{\frac{1}{2}}\times e^{j2\pi.\left(\left(1-\left(\frac{\gamma_{\tilde{A}}}{2\pi}\right)^{2}\right)^{\lambda}-\left(1-\left(\frac{\gamma_{\tilde{A}}}{2\pi}\right)^{2}-\left(\frac{\beta_{\tilde{A}}}{2\pi}\right)^{2}\right)^{\lambda}\right)^{\frac{1}{2}}}\Bigg\},\lambda>0 \tag{17}$$

$$\widetilde{A}_{\varsigma}\oplus\widetilde{B}_{\varsigma}=\Bigg\{\left(\varsigma_{\mu_{\tilde{A}}}{}^{2}+\varsigma_{\mu_{\tilde{B}}}{}^{2}-\varsigma_{\mu_{\tilde{A}}}{}^{2}\varsigma_{\mu_{\tilde{B}}}{}^{2}\right)^{\frac{1}{2}}\times e^{j2\pi.\left(\left(\frac{\alpha_{\tilde{A}}}{2\pi}\right)^{2}+\left(\frac{\alpha_{\tilde{B}}}{2\pi}\right)^{2}-\left(\frac{\alpha_{\tilde{A}}}{2\pi}\right)^{2}\left(\frac{\alpha_{\tilde{B}}}{2\pi}\right)^{2}\right)^{\frac{1}{2}}},\varsigma_{\nu_{\tilde{A}}}\varsigma_{\nu_{\tilde{B}}}\times e^{j2\pi.\left(\left(\frac{\gamma_{\tilde{A}}}{2\pi}\right)\left(\frac{\gamma_{\tilde{B}}}{2\pi}\right)\right)},\left(\left(1-\varsigma_{\mu_{\tilde{B}}}{}^{2}\right)\varsigma_{h_{\tilde{A}}}{}^{2}+\left(1-\varsigma_{\mu_{\tilde{A}}}{}^{2}\right)\varsigma_{h_{\tilde{B}}}{}^{2}-\varsigma_{h_{\tilde{A}}}{}^{2}\varsigma_{h_{\tilde{B}}}{}^{2}\right)^{\frac{1}{2}}\times e^{j2\pi.\left(\left(1-\left(\frac{\alpha_{\tilde{B}}}{2\pi}\right)^{2}\right)\left(\frac{\beta_{\tilde{A}}}{2\pi}\right)^{2}+\left(1-\left(\frac{\alpha_{\tilde{A}}}{2\pi}\right)^{2}\right)\left(\frac{\beta_{\tilde{B}}}{2\pi}\right)^{2}-\left(\frac{\beta_{\tilde{A}}}{2\pi}\right)^{2}\left(\frac{\beta_{\tilde{B}}}{2\pi}\right)^{2}\right)^{\frac{1}{2}}}\Bigg\} \tag{18}$$

$$\widetilde{A}_{\varsigma}\otimes\widetilde{B}_{\varsigma}=\Bigg\{\varsigma_{\mu_{\tilde{A}}}\varsigma_{\mu_{\tilde{B}}}e^{j2\pi.\left(\frac{\alpha_{\tilde{A}}}{2\pi}\right)\left(\frac{\alpha_{\tilde{B}}}{2\pi}\right)},\left(\varsigma_{\nu_{\tilde{A}}}{}^{2}+\varsigma_{\nu_{\tilde{B}}}{}^{2}-\varsigma_{\nu_{\tilde{A}}}{}^{2}\varsigma_{\nu_{\tilde{B}}}{}^{2}\right)^{\frac{1}{2}}\times e^{j2\pi.\left(\left(\frac{\gamma_{\tilde{A}}}{2\pi}\right)^{2}+\left(\frac{\gamma_{\tilde{B}}}{2\pi}\right)^{2}-\left(\frac{\gamma_{\tilde{A}}}{2\pi}\right)^{2}\left(\frac{\gamma_{\tilde{B}}}{2\pi}\right)^{2}\right)^{\frac{1}{2}}},\left(\left(1-\varsigma_{\nu_{\tilde{B}}}{}^{2}\right)\varsigma_{h_{\tilde{A}}}{}^{2}+\left(1-\varsigma_{\nu_{\tilde{A}}}{}^{2}\right)\varsigma_{h_{\tilde{B}}}{}^{2}-\varsigma_{h_{\tilde{A}}}{}^{2}\varsigma_{h_{\tilde{B}}}{}^{2}\right)^{\frac{1}{2}}\times e^{j2\pi.\left(\left(1-\left(\frac{\gamma_{\tilde{B}}}{2\pi}\right)^{2}\right)\left(\frac{\beta_{\tilde{A}}}{2\pi}\right)^{2}+\left(1-\left(\frac{\gamma_{\tilde{A}}}{2\pi}\right)^{2}\right)\left(\frac{\beta_{\tilde{B}}}{2\pi}\right)^{2}-\left(\frac{\beta_{\tilde{A}}}{2\pi}\right)^{2}\left(\frac{\beta_{\tilde{B}}}{2\pi}\right)^{2}\right)^{\frac{1}{2}}}\Bigg\} \tag{19}$$

3.2. The extension of DEMATEL

The method of DEMATEL (Decision Making and Trial Evaluation Laboratory) was introduced in the 1970s to measure the criteria interactively in the decision-making process. The method has the most important advantages with the computations of the impact-relation directions for the attributes. Thus, the possible influences can be obtained comprehensively to solve complex relations between the criteria. In the last decades, the extensions of DEMATEL have been generated to proceed with the robustness of the methodology for complicated real-world problems [90–92]. The extension with the quantum spherical fuzzy sets can be proposed as follows:

Step 1: Dependency degrees among the criteria are determined by the decision-makers. Linguistic evaluations are collected for constructing the relation matrix.

Step 2: Quantum spherical fuzzy relation matrix is defined by considering the linguistic evaluations of decision makers and the quantum spherical fuzzy numbers. $\varsigma_{\mu_{\tilde{A}}}$ $\varsigma=\left[\varsigma_{ij}\right]_{n\times n}$ is the relationship degrees of criteria on the other criteria. ς_{ij} gives information on the influence degree of criterion *i* stated in the row over criterion j given in the column. The matrix is illustrated in Eq. (20)

$$\varsigma_{k}=\begin{bmatrix}0 & \varsigma_{12} & \cdots & \cdots & \varsigma_{1n}\\ \varsigma_{21} & 0 & \cdots & \cdots & \varsigma_{2n}\\ \vdots & \vdots & \ddots & \cdots & \cdots\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ \varsigma_{n1} & \varsigma_{n2} & \cdots & \cdots & 0\end{bmatrix} \tag{20}$$

where ςk defines the quantum spherical fuzzy direct relation matrix (A). $\varsigma_{ij}=\left(\varsigma_{\mu_{ij}}e^{2\pi.\alpha_{ij}},\varsigma_{\nu_{ij}}e^{2\pi.\gamma_{ij}},\varsigma_{h_{ij}}e^{2\pi.\beta_{ij}}\right)$ and k is the number of decision-makers.

The aggregated values ς of the decision makers are computed in the form of quantum spherical fuzzy numbers by Eq. (21) [88]:

$$\varsigma=\Bigg\{\left[1-\prod_{i=1}^{k}\left(1-\varsigma_{\mu_i}{}^{2}\right)^{\frac{1}{k}}\right]^{\frac{1}{2}}\times e^{2\pi.\left[1-\prod_{i=1}^{k}\left(1-\left(\frac{\alpha_i}{2\pi}\right)^{2}\right)^{\frac{1}{k}}\right]^{\frac{1}{2}}},\prod_{i=1}^{k}\varsigma_{\nu_i}{}^{\frac{1}{k}}\times e^{2\pi.\prod_{i=1}^{k}\left(\frac{\gamma_i}{2\pi}\right)^{\frac{1}{k}}},\left[\prod_{i=1}^{k}\left(1-\varsigma_{\mu_i}{}^{2}\right)^{\frac{1}{k}}-\prod_{i=1}^{k}\left(1-\varsigma_{\mu_i}{}^{2}-\varsigma_{h_i}{}^{2}\right)^{\frac{1}{k}}\right]^{\frac{1}{2}}\times e^{2\pi.\left[\prod_{i=1}^{k}\left(1-\left(\frac{\alpha_i}{2\pi}\right)^{2}\right)^{\frac{1}{k}}-\prod_{i=1}^{k}\left(1-\left(\frac{\alpha_i}{2\pi}\right)^{2}-\left(\frac{\beta_i}{2\pi}\right)^{2}\right)^{\frac{1}{k}}\right]^{\frac{1}{2}}}\Bigg\} \tag{21}$$

Step 3: The defuzzified values $Def\varsigma$ of quantum spherical fuzzy sets are computed using the score function by Eq. (22) [89].

$$Def\varsigma_i=\varsigma_{\mu_i}+\varsigma_{h_i}\left(\frac{\varsigma_{\mu_i}}{\varsigma_{\mu_i}+\varsigma_{\nu_i}}\right)+\left(\frac{\alpha_i}{2\pi}\right)+\left(\frac{\gamma_i}{2\pi}\right)\left(\frac{\left(\frac{\alpha_i}{2\pi}\right)}{\left(\frac{\alpha_i}{2\pi}\right)+\left(\frac{\beta_i}{2\pi}\right)}\right) \tag{22}$$

Step 4: The direct relation matrix is normalized. The normalized direct relationship matrix $B=\left[b_{ij}\right]_{n\times n}$ is given by Eq. (23):

Table 12
Score function of the alternatives for quantum spherical fuzzy sets.

Criteria/ Alternatives	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	1.236	1.259	1.236	1.284	1.297	1.236	1.236	1.263	1.236	1.300
A2	1.245	1.236	1.243	1.236	1.285	1.245	1.236	1.263	1.236	1.236
A3	1.243	1.297	1.297	1.236	1.236	1.236	1.245	1.236	1.300	1.245
A4	1.266	1.266	1.236	1.236	1.236	1.236	1.236	1.236	1.236	1.236
A5	1.236	1.263	1.297	1.236	1.247	1.266	1.297	1.247	1.236	1.266
A6	1.236	1.266	1.300	1.236	1.297	1.236	1.263	1.236	1.236	1.266
A7	1.236	1.236	1.300	1.236	1.236	1.236	1.236	1.236	1.245	1.297
A8	1.236	1.297	1.247	1.243	1.245	1.236	1.236	1.236	1.245	1.236
A9	1.263	1.236	1.263	1.247	1.236	1.236	1.247	1.263	1.243	1.247
A10	1.263	1.300	1.269	1.236	1.236	1.236	1.263	1.236	1.236	1.236

Table 13
Normalized decision matrix.

Criteria/ Alternatives	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	0.314	0.315	0.308	0.327	0.327	0.315	0.313	0.321	0.314	0.327
A2	0.316	0.309	0.310	0.315	0.324	0.318	0.313	0.321	0.314	0.311
A3	0.315	0.324	0.323	0.315	0.311	0.315	0.315	0.314	0.330	0.313
A4	0.321	0.316	0.308	0.315	0.311	0.315	0.313	0.314	0.314	0.311
A5	0.314	0.315	0.323	0.315	0.314	0.323	0.328	0.317	0.314	0.318
A6	0.314	0.316	0.324	0.315	0.327	0.315	0.320	0.314	0.314	0.318
A7	0.314	0.309	0.324	0.315	0.311	0.315	0.313	0.314	0.316	0.326
A8	0.314	0.324	0.311	0.316	0.314	0.315	0.313	0.314	0.316	0.311
A9	0.320	0.309	0.315	0.317	0.311	0.315	0.316	0.321	0.316	0.314
A10	0.320	0.325	0.316	0.315	0.311	0.315	0.320	0.314	0.314	0.311

Table 14
Weighted decision matrix.

Criteria/ Alternatives	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	0.031	0.031	0.031	0.033	0.033	0.032	0.031	0.032	0.031	0.033
A2	0.032	0.031	0.031	0.031	0.032	0.032	0.031	0.032	0.031	0.031
A3	0.032	0.032	0.032	0.031	0.031	0.032	0.032	0.031	0.033	0.031
A4	0.032	0.032	0.031	0.031	0.031	0.032	0.031	0.031	0.031	0.031
A5	0.031	0.032	0.032	0.031	0.031	0.032	0.033	0.032	0.031	0.032
A6	0.031	0.032	0.032	0.031	0.033	0.032	0.032	0.031	0.031	0.032
A7	0.031	0.031	0.032	0.031	0.031	0.032	0.031	0.031	0.032	0.033
A8	0.031	0.032	0.031	0.032	0.031	0.032	0.031	0.031	0.032	0.031
A9	0.032	0.031	0.031	0.032	0.031	0.032	0.032	0.032	0.032	0.031
A10	0.032	0.032	0.032	0.031	0.031	0.032	0.032	0.031	0.031	0.031

Table 15
Ranking results.

Alternatives	D+	D-	RCi	Ranking
A1	0.003	0.003	0.463	4
A2	0.004	0.001	0.276	8
A3	0.003	0.003	0.478	1
A4	0.004	0.001	0.209	10
A5	0.003	0.003	0.470	3
A6	0.003	0.003	0.473	2
A7	0.004	0.002	0.388	5
A8	0.004	0.002	0.305	7
A9	0.004	0.001	0.263	9
A10	0.003	0.002	0.376	6

$$B = \frac{\varsigma}{max_{1 \le i \le n} \sum_{j=1}^{n} \varsigma_{ij}} \tag{23}$$

where,

$$\text{where}, 0 \le b_{ij} \le 1 \tag{24}$$

Step 5: The total relation matrix is constructed. The total relation matrix $C = [c_{ij}]_{n \times n}$ is defined as follows [88]:

$$\lim_{k \to \infty} (B + B^2 + \cdots + B^k) = B(I - B)^{-1} \tag{25}$$

where I is the identity matrix. The total relation matrix gives information about the influence of ith criterion on the j^{th} criterion denoted by e_{ij}.

Step 6: The total cause and effect are computed. The cause factors $D = [d_{ij}]_{n \times 1}$ are listed with the sums of rows as the effect factors $E = [e_{ij}]_{1 \times n}$ are given in the sums of columns by Eqs. (26) and (27).

$$D = \left[\sum_{j=1}^{n} e_{ij} \right]_{nx1} \tag{26}$$

$$E = \left[\sum_{i=1}^{n} e_{ij} \right]_{1xn} \tag{27}$$

The values of (D+E) show the relative importance of the criteria. However, the values of (D-E) define the influence directions among the criteria.

The impact-relation directions are represented by using threshold value α as follows:

Table 16
Comparative results with sensitivity analysis.

Alternatives	Case 1		Case 2		Case 3		Case 4		Case 5	
	TOPSIS	VIKOR	TOPSIS	VIKOR	TOPSIS	VIKOR	TOPSIS	VIKOR	TOPSIS	VIKOR
A1	4	4	4	4	4	4	4	4	4	4
A2	8	8	8	8	8	8	8	8	8	8
A3	1	1	1	1	1	1	1	1	1	1
A4	10	10	10	10	10	9	10	9	10	9
A5	3	3	3	3	3	3	3	3	3	3
A6	2	2	2	2	2	2	2	2	2	2
A7	5	5	5	5	5	5	5	5	5	5
A8	7	7	7	7	7	7	7	7	7	7
A9	9	9	9	9	9	10	9	10	9	10
A10	6	6	6	6	6	6	6	6	6	6
Alternatives	**Case 6**		**Case 7**		**Case 8**		**Case 9**		**Case 10**	
	TOPSIS	VIKOR	TOPSIS	VIKOR	TOPSIS	VIKOR	TOPSIS	VIKOR	TOPSIS	VIKOR
A1	4	4	4	4	4	4	4	4	4	4
A2	8	8	8	8	8	8	8	8	8	8
A3	1	1	1	1	1	1	1	1	1	1
A4	10	10	10	10	10	10	10	10	10	10
A5	3	3	3	3	3	3	3	3	3	3
A6	2	2	2	2	2	2	2	2	2	2
A7	5	5	5	5	5	5	5	5	5	5
A8	7	7	7	7	7	7	7	7	7	7
A9	9	9	9	9	9	9	9	9	9	9
A10	6	6	6	6	6	6	6	6	6	6

$$\alpha = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}[e_{ij}]}{N} \tag{28}$$

where N is the total number of criteria in the matrix. It is assumed that if a criterion in the row has a higher value than the threshold, it affects the criterion of the column in the matrix.

3.3. The extension of TOPSIS

TOPSIS (Technique of Order Preference Similarity to the Ideal) method was first studied by [93,94] in the 1980s and it has an important advantage for ranking the alternatives based on the distances to the positive and negative ideal solutions. In this study, we propose an extension of TOPSIS based on QSFNs, one of the most recent extensions for providing more comprehensive results than the conventional multicriteria decision-making approaches in complex decision-making problems. The proposed model is given as follows:

Step 1: Linguistic evaluations are collected for constructing the decision matrix.

Step 2: Quantum spherical fuzzy decision matrix is defined by considering the linguistic evaluations of decision makers based on QSFNs. $X = [X_{ij}]_{n\times m}$ is the decision matrix. X_{ij} gives information on alternative i with respect to criterion j stated. The matrix is illustrated in Eq. (29).

$$X_k = \begin{bmatrix} 0 & X_{12} & \cdots & \cdots & X_{1m} \\ X_{21} & 0 & \cdots & \cdots & X_{2m} \\ \vdots & \vdots & \ddots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ X_{n1} & X_{n2} & \cdots & \cdots & 0 \end{bmatrix} \tag{29}$$

where X defines the quantum spherical fuzzy decision matrix. $X_{ij} = \left(X_{\mu_{ij}} e^{2\pi.\alpha_{ij}}, X_{v_{ij}} e^{2\pi.\gamma_{ij}}, X_{h_{ij}} e^{2\pi.\beta_{ij}}\right)$ and k is the number of decision-makers.

Similarly, the aggregated values X of decision makers are calculated by Eq. (21).

Step 3: The defuzzified values of the quantum spherical fuzzy sets for the decision matrix are constructed by using the score function via Equation (22).

Step 4: The normalized values using vector normalization procedure are defined by Eq. (30):

$$r_{ij} = \frac{X_{ij}}{\sqrt{\sum_{i=1}^{m} X_{ij}^2}}. \tag{30}$$

Step 5: The values are weighted based on Eq. (31):

$$v_{ij} = w_{ij} \times r_{ij} \tag{31}$$

Step 6: The positive (A^+) and negative (A^-) ideal solutions are defined as follows:

$$A^+ = \left\{v_{1j}, v_{2j}, \ldots, v_{mj}\right\} = \left\{maxv_{1j}\ for\ \forall j \in n\right\}, \tag{32}$$

$$A^- = \left\{v_{1j}, v_{2j}, \ldots, v_{mj}\right\} = \left\{minv_{1j}\ for\ \forall j \in n\right\}. \tag{33}$$

Step 7: The distances to the best (D_i^+) and worst alternatives (D_i^-) for each criterion are computed by using Eqs. (34) and (35):

$$D_i^+ = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - A_j^+\right)^2}, \tag{34}$$

$$D_i^- = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - A_j^-\right)^2}. \tag{35}$$

Step 8: The relative closeness to the ideal solutions (RC_i) is calculated by Eq. (36):

$$RC_i = \frac{D_i^-}{D_i^+ + D_i^-}. \tag{36}$$

4. Analysis

This study provides a new hybrid model based on quantum spherical fuzzy sets. For this purpose, a two-stage decision-making approach is used to measure the performance of the sector alternatives in terms of the ESG Index. In the first stage, quantum spherical fuzzy DEMATEL is employed to figure out the impact-relation directions and the weights of the criteria. In the second stage, the extended TOPSIS with the quantum spherical fuzzy sets is considered to rank the sector alternatives for the ESG Index performance. The flowchart of the proposed model and

Table A.1
Summary of the literature review in emerging markets.

Reference	Subject	Components & Scope	Methods/Approach	Findings
[67]	ESG Disclosures	568 firm-year observations from 78 (PLCs) in Malaysia	Panel Regression Analysis	ESG disclosure scores are significantly improved with the increasing presence of women directors on corporate boards
[99]	Gender Diversity & Corporate Sustainability Disclosure	878 PLCs in 11 Industries in Malaysia	Panel Regression Analysis	Board gender diversity increases corporate sustainability disclosures
[100]	Board Diversity & Sustainability performance	100 PLCs in Malaysia; Environmental Reporting; Firms Characteristics; Diversity; Gender and Religious	Hierarchical Tobit Regression with robust standard error	Board diversity (on the religious difference), firm size, profitability, and growth have significant influence on environmental disclosures
[101]	ESG & Firm Value	Sample of 245 Thai non-financial listed companies	Regression Analysis	Neither independent directors nor grey directors are the significant factors in improving firm value.
[102]	Corporate Governance & Firm Performance	493 firms of non-financial firms in Thailand	Panel Regression Analysis	Corporate governance does not affect financial leverage and firm performance.
[103]	Board Diversity & Firm Performance	3,876 public firms in 47 countries	Regression Analysis	Independent directors do not contribute to firm performance unless the board is gender diversified, Female directors enhance board effectiveness.
[66]	ESG & CSR	Firm-Specific Variables; CSR; 100 PLCs in Malaysia, Pakistan, and Thailand	Panel OLS Multiple Regression Analysis	There is a significant relationship between board gender diversity and enhanced adoption of CSR in Asia-Pacific Countries
[104]	Board Diversity & Firm Performance in Southern Asia	308 firm observations from Hong Kong, 2941 from South Korea, 1241 from Malaysia and 1013 from Singapore	Panel Regression Analysis	Increasing number of female directors on the board has a positive effect on firm performance
[64]	Board Structure & ESG Disclosure in Latin America	306 PLCs in Latin American Countries; 704 firm-year observations; ESG, Board Size; Gender Diversity; CEO Duality	Panel Regression Analysis	Board size and independent directors positively impact ESG disclosure, but women on the board and CEO duality impact ESG disclosure negatively.
[105]	ESG; Institutional investor preferences in Latin America	439 non-financial firms from Latin American countries (Argentina, Brazil, Chile, Colombia, Mexico, and Peru) from 2001 to 2011. 4,399 firm-year observations	Panel Regression Analysis	Institutional investors prefer experience and education, while dislike CEO entrenchment. Independent institutional investors value more directors' professional experience.
[68]	ESG and Financial Performance of PLCs in Latin America	104 PLCs from Brazil, Chile, Colombia, Mexico, and Peru from 2011 to 2015	Panel Regression Analysis	The relationship between the ESG score and FP is significantly negative
[106]	ESG and Board Diversity in Korea	Korean Data	Quantitative and Qualitative Analysis	This study indicates that companies invested in the Women Fund had a higher return on assets and return on equity.
[69]	ESG performance and Firm Value in Korea	KEJI index that covers 200 firms	Panel Regression Analysis	CSR practices positively and significantly affect firm's market value.
[70]	Board Composition & ESG	ESG scores of PLCs in KSA, BAE, Kuwait, Qatar, Bahrain, Oman between 2008 and 2017	Panel Regression Analysis	Higher board independence and female board participation facilitate the transmission of a firm's positive image by improving social responsibility
[107]	Board Diversity and CSR performance	117 company reports for 500 biggest companies in Turkey.	Panel Regression Analysis	CSR performance increases when there is a more presence of independent board members in a firm; Female and foreign board members do not have any significant effect on CSR performance
[108]	CSR, Ownership Structure, and Firm Performance	Non-financial Turkish companies listed in the Borsa Istanbul Sustainability Index; CSP throughout 2014–2018	Panel Regression Analysis	There is a positive influence of foreign and institutional ownerships on CSP. The CSP is also positively correlated with the board size and the proportion of independent board members.
[109]	Board Diversity & Firm Performance	Data from 2008–2012 of the PLCs in Turkey	Panel Regression Analysis	The presence of female directors is positively related to firms' financial performance.

computation results are given in Fig. 1:

Stage 1: Weighting the criteria for the ESG Index

Step 1: Determine the criteria for the ESG Index

In Table 1, the dimensions and criteria are defined for the ESG Index.

Step 2: Collect the linguistic evaluations of decision makers for the criteria.

Three decision makers have been selected to collect their linguistic evaluations on the relation matrix of the criteria. They are experts with at least fifteen-year experience in the field of corporate governance and strategic decision-making. In this process, firstly, the questions are created by comparing these selected criteria. For the criteria list, 90 different questions are generated. Within this framework, online meetings are conducted with the experts. In these meetings, the evaluations of the experts for these questions are collected. They shared their linguistic evaluations by using the scales in Table 2. The evaluation results are given in Table 3.

Step 3: Determine the average values of quantum spherical fuzzy numbers for the criteria.

The average values of quantum spherical fuzzy numbers are computed by using Eq. (21) and the results are given in Table 4.

Step 4: Compute the score function of the criteria for quantum spherical fuzzy sets.

In the proposed methodology, the defuzzification process serves as a crucial step in transforming the quantum spherical fuzzy numbers, which represent the decision matrix, into real numbers. This transformation allows for further analysis and comparison of alternatives based on their proximity to ideal solutions, facilitating a more comprehensive evaluation of complex decision-making problems. The defuzzification process, as described in Eq. (22), employs a score function to convert each element of the quantum spherical fuzzy decision matrix, denoted as X_{ij}, into a crisp, real-valued quantity referred to as $DefX_{ij}$. This transformation is accomplished by considering the linguistic evaluations provided by decision-makers and leveraging the parameters (μ, α, γ, β) associated with the quantum spherical fuzzy number.

This transformation is critical for subsequent analysis, as it enables the comparison and ranking of alternatives based on their distances to the ideal solutions. Hence, the defuzzification process in the proposed methodology acts as a vital bridge between the realm of quantum spherical fuzzy numbers and the domain of real numbers. It facilitates the conversion of intricate and ambiguous linguistic evaluations into precise and tangible numerical values, thereby enhancing the effectiveness and robustness of the decision-making process. The defuzzified values of the criteria are computed by the score function of the fuzzy sets as in Eq. (22). The score function values are presented in Table 5.

Step 5: Calculate the normalized relation matrix.

The normalization process is applied for the relation matrix according to Eqs. (23) and (24). The normalized values are presented in Table 6.

Step 6: Construct the total relation matrix and the directions among the criteria.

In the first stage of our proposed methodology, we compute the directions of the criteria as well as their weights to gain a comprehensive understanding of the impact relationships among the different ESG Index components. To achieve this, we construct the total relation matrix and determine the impact directions among the criteria using Eqs. (25)–(28). The results of this computation are presented in Table 7.

The average value of the total relation matrix is considered a threshold value. A higher value than the threshold shows that the criterion at the row has an impact on the criterion at the column of the relation matrix. By using Eq. (28), the threshold value is computed as 7.412. According to this assumption, it seems that Innovation (C3) affects all remaining ESG Index components. H2O emissions (C1) has also an important effect on Resource Usage, Innovation, Community Investment, Product Responsibility, Shareholder Rights, Management Structure, and Compensation, respectively. Gender Equity (C5) has significant impact relations on Resource Usage (C2), Innovation (C3), Community Investment (C4), Product Responsibility (C6), Shareholder Rights (C8), Management Structure and Compensation (C9), and CSR Strategy (C10).

Shareholder Rights (C8), Management Structure and Compensation (C9), and Product Responsibility (C6) components of the ESG scores with their impact relation scores, only have impact relations on several ESG components. Shareholders' Rights (C8) have a significant impact relations with only Resource Usage (C2), whereas Product Responsibility (C6) has significant impact relations with only Resource Usage (C2), Innovation (C3), and Management Structure and Compensation (C9) of the ESG scores.

Step 7: Compute the influence degrees and the weights of the criteria.

Table 8 shows the total effects and directions of the criteria as well as the weighting priorities. In this framework, Eqs. (26) and (27) are taken into consideration. For this purpose, the normalized values of (D+E) are considered to define the weights of the criteria.

In Table 8, H20 Emissions, Innovation, Community Investment, Gender Equity, Human Rights, and CSR Strategy (C1, C3, C4, C5, C7, and C10) are listed as the most influencing components of the ESG Index with their impact relations scores, while the remaining components, i.e., Resource Usage, Product Responsibility, Shareholders' Rights (C2, C6, C8, and C9), are under the influence of a remaining set of the ESG Index criteria with their negative values of (D-E).

Furthermore, Innovation (C3) and Resource Usage (C2) among the other ESG Index components have the highest priority scores that determine the level of importance and weight for all sector alternatives from financial services (A1) to utilities sector (A10). Surprisingly, Human Rights (C7) and Shareholder's Rights (C8) have the lowest priority scores. From Table 7, Human Rights (C7) component as an influencing attribute has impact relations with Resource Usage (C2), Innovation (C3), Product Responsibility (C6), Management Structure, and Compensation (C9) even though it has the lowest priority score.

Stage 2: Ranking the ESG Index performances of the sector alternatives for emerging markets.

Step 1: Collect the linguistic evaluations of decision-makers for the sector alternatives.

The decision-makers provide their linguistic evaluations to evaluate the performances of the sector alternatives in emerging economies with respect to the TR cumulative ESG Index scores of companies operating in each sector. For this purpose, 10 sector alternatives are selected using one of *Yahoo Finance*'s most popular databases. Yahoo Finance categorizes the sectors based on their importance for effective econometric analysis. Therefore, our experts used Yahoo Finance's sector alternatives before making their evaluation. Table 9 shows the sector alternatives: Financial Services (A1), Consumer Defensive (A2), Technology (A3), Real Estate (A4), Energy (A5), Communication Services (A6), Healthcare (A7), Consumer Cyclicals (A8), Basic Materials (A9) and Utilities (A10). In Table 10, the decision makers provide their evaluations for the alternatives on the ESG Index by using the linguistic scales in Table 2.

Step 2: Define the average values of quantum spherical fuzzy numbers for the alternatives.

Similarly, the average values of the fuzzy numbers are computed for the alternatives with Eq. (21). The results are given in Table 11.

Step 3: Compute the score function of the alternatives for quantum spherical fuzzy sets.

By using Eq. (22), the defuzzified values of the alternatives are presented as the score function in Table 12.

Step 4: Normalize the decision matrix.

Normalization procedure is employed by Eq. (30). Table 13 shows the normalized values of the decision matrix.

Step 5: Compute the weighted decision matrix.

The weighted decision matrix is computed by Eq. (31). Table 14 gives the weighted decision matrix.

Step 6: Rank the alternatives.

In the final step, the values of RC_i are calculated by Eqs. (32)–(36). Table 15 shows the final ranking results according to the descending order of the RC_i values.

The ranking results show that Technology (A3) has the best rank, while Communication Services (A6) and Energy (A5) sectors come in the second and third ranks, respectively. Utilities (A4) has the worst ranking in terms of the ESG performance scores, whereas Basic Materials (A9), and Consumer Defensive (A2) are the other worst-ranking sector alternatives with their cumulative and directional impact relations of ESG Index scores. A4 has the worst performance in the ESG Index performance of the sector alternatives in emerging markets.

However, the comparative methodology with the sensitivity analysis is also applied for the robustness check and the coherency of the proposed model. For that, the extended VIKOR method is considered to compare the ranking results. For the sensitivity analysis, the weighting results are changed consecutively with 10 Cases and the results are given in Table 16.

The alternatives (A1 to A10) represent the entities or options being evaluated in the decision-making process. Each case represents a different ablation scenario, where the criteria weights have been changed consecutively. The TOPSIS and VIKOR rankings or scores for each alternative under each case reflect the performance or evaluation results based on the modified weights. According to the comparative ranking results with sensitivity analysis, the ranking results of the extended TOPSIS and VIKOR are almost same for each case. This analysis provides valuable insights into the sensitivity of the evaluation results to changes in the importance of different criteria. These results also demonstrate that the ranking results of the extended TOPSIS methodology are coherent and applicable for the different applications and other possible extensions of the decision-making approaches.

The comparative analysis results presented in Table 16 exhibit a remarkable coherence, laying a solid foundation for further extensions of fuzzy decision-making modeling. The consistent rankings across different ablation scenarios demonstrate the robustness and adaptability of the proposed model based on quantum spherical fuzzy sets. This coherence suggests that the model can withstand changes in criteria weights, thus providing reliable evaluations in diverse decision scenarios. Although the extended TOPSIS and VIKOR rankings show a high level of similarity in the sensitivity analysis, it is essential to emphasize that our proposed model delivers superior results compared to the conventional use of TOPSIS and VIKOR. The advantages lie in the integration of the phase angles within quantum spherical fuzzy sets, which bestows the model with the ability to provide a more comprehensive and accurate representation of uncertainties, hesitancy, and complex information sets. By considering the relative positioning and phase relationships of the membership, non-membership, and hesitancy degrees, our proposed model can offer deeper insights and finer-grained evaluations in decision-making processes.

The performance of the proposed model is evident in the comparative results, where the consistent and coherent rankings across different cases showcase its robustness and applicability in a variety of decision-making scenarios. It is worth noting that even with similar rankings to the extended VIKOR, the proposed model excels in providing more accurate and nuanced evaluations, elevating the decision-making process to a new level of sophistication. The incorporation of quantum spherical fuzzy sets and phase angles empowers the model with a comprehensive framework for handling uncertainties and vagueness in decision-making. This ability to quantify uncertainties enhances the model's reliability and practicality in real-world applications.

By utilizing phase angles to represent probabilities, the proposed model offers enhanced precision in ranking and evaluating alternatives, leading to more informed and effective decision-making outcomes. The innovative integration of fuzzy logic and quantum mechanics in the proposed model bridges the gap between classical and quantum decision-making paradigms, providing a novel approach with unique insights into uncertainty management. The model's ability to encompass multi-criteria decision analysis, resource allocation, risk management, project selection, and supply chain optimization demonstrates its versatility and wide-ranging applicability across various domains. The coherent results obtained from sensitivity analysis demonstrate the robustness and stability of the proposed model under varying criteria weights, ensuring consistent evaluations in dynamic decision scenarios.

5. Discussions

Even though Gender Equity (C5), Management Structure (C9), and CSR Strategy (C10) enhance the sectorial performance of companies, the findings shed light on the fact that all ESG components have different and directional impact relation levels. In our analysis, Gender Equity has strong directional impact relations on Resource Usage (C2), Innovation (C3), Community Investment (C4), Product Responsibility (C6), Shareholder Rights (C8), Management Structure (C9), and CSR Strategy. Companies should pay more attention to board diversity and gender equity to achieve effective sustainability performance. Sectors that have companies with strong ESG scores will attract more socially and environmentally conscious investors.

The findings also highlight that H20 Emissions (C1), Innovation (C3), Community Investment (C4), Gender Equity (C5), Human Rights (C7), and CSR Strategy (C10) components of the ESG Index are listed as the most influencing components with their directional impact relations scores. Future research on the ESG Index variables may examine the impact of C1, C3, C4, C5, and C7 on other components, i.e., Resource Usage (C2), Product Responsibility (C6), Shareholders' Rights (C8), Management Structure (C9). Furthermore, the results show that Innovation (C3) and Resource Usage (C2) among the other ESG Index components have the highest priority scores that determine the level of importance and weight for all sector alternatives. Companies in Technology Sector (A3), Communication Services (A6), and Energy Sector (A5) exhibit good ESG performance, relying on Innovation (C3) and Resource Usage (C2) practices. Low-performing companies in Utilities (A4), Basic Materials (A9) and Consumer Defensive (A2) sectors should enhance their ESG Index scores with well-crafted Innovation (C3) and efficient Resource Usage (C2) practices.

The results are expected to have a more directional impact-relations performance of Human Rights components (C7) of the ESG Index and Shareholder's Rights (C8) for all sectors. However, our analysis shows that Human Rights and Shareholders' Rights have the lowest priority scores. Nevertheless, Human Rights (C7) as an influencing attribute has an impact on relations with Resource Usage (C2), Innovation (C3), Product Responsibility (C6), and Management Structure and Compensation (C9) even it has the lowest priority score. Companies with high ESG scores in Technology (A3), Telecommunication (A6), and Energy (A5) sectors should continue increasing the performance scores on human rights and shareholders' activities.

Innovation plays an important role in achieving the sustainability goals of businesses. First, innovation is vital in developing innovative solutions to reduce environmental impacts. In other words, innovation is essential for the development of environmentally friendly products. In this way, businesses will consume less natural resources and thus, sustainability targets will be achieved more easily. Innovation is also essential for achieving economic sustainability goals [95]. Thanks to innovation, businesses can reach new customers much more easily. This situation contributes to the increase of the competitiveness of the enterprises. On the other hand, with the effect of globalization, competition has increased significantly in most of the sectors. Developing products based on innovation helps customers to meet their expectations more easily in this process. Customers who are satisfied with the services will continue to work with these companies [96]. This is an important issue for businesses to achieve their long-term sustainability goals. Li et al. [97] and Dong et al. [98] also underlined the importance of the innovation in this context.

Our proposed decision-making method has demonstrated remarkable capabilities in providing more accurate and reasonable output compared to existing approaches. The results obtained through the utilization of the proposed methodology showcase its superior performance in effectively evaluating the ten criteria and ten alternatives. By incorporating the principles of quantum mechanics, fuzzy sets, and the golden cut, our method surpasses traditional decision-making approaches in comprehensively capturing nuanced degrees of membership, non-membership, and hesitancy, thus yielding more precise and reliable results. In the comparative analysis with sensitivity, the proposed extended TOPSIS and VIKOR rankings exhibit remarkable coherence across multiple ablation scenarios, reaffirming the robustness and applicability of our methodology. Furthermore, by incorporating the phase angle as a pivotal parameter within the realm of quantum spherical fuzzy sets, our approach unveils unique insights into the interplay and relative importance of different components, fostering a deeper exploration of uncertainty and decision-making problems.The implications of our proposed method extend beyond the evaluation of the ten criteria and ten alternatives in this specific study. The integration of quantum mechanics and fuzzy sets equips decision-makers with a powerful tool for handling complex real-world problems across diverse domains. Our methodology enhances the understanding of uncertainty and empowers the decision-making process in scenarios where qualitative and indefinite evaluations abound. By considering multiple parameters simultaneously, our comprehensive approach provides a holistic grasp of uncertain information, allowing for more informed and well-balanced decisions. Moreover, our proposed method builds upon established theoretical foundations, incorporating the rigor of quantum mechanics and the efficiency of fuzzy sets. This ensures the credibility and reliability of the results, offering an innovative and robust solution for tackling intricate decision-making challenges. The integration of the golden cut further enhances the accuracy of decision-making under uncertainty, defining optimal ratios among the scales of spherical fuzzy sets and illuminating geometric patterns in complex decision problems.

The main benefit of this model is that DEMATEL technique is used to weight the criteria. Because of considering causal directions, more effective findings can be reached [27,28]. This situation can be accepted as the main superiority of this proposed model. However, multi-SWARA model can also be used for this purpose according to many different scholars since this methodology can also use impact direction map [110–112]. Moreover, using Spherical fuzzy sets also provide some benefits because they consider a wider data set. This situation helps to reach more appropriate results. However, Pythagorean fuzzy sets were also preferred in many different studies in this respect [113,114].

6. Conclusions

In the study, we developed a hybrid business analytics model that employs a golden cut-enhanced quantum spherical fuzzy decision-making approach. We have found that Innovation (C3) and Resource Usage (C2) among the other ESG Index components have the highest priority scores that determine the level of importance and weight for all sector alternatives from Financial Services (A1) to Utilities sector (A10). Thus, based on the results, the significance of innovation and resource usage indexes on the other ESG components necessitates the importance of managers in technology and communication industries to augment their digital balance sheet to invest in more innovative technologies that favor green production to remain competitive and attract more potential investors. We acknowledge that this study has some limitations. First, it employs a hybrid business analytics model blending expert opinions with quantitative data. Future research should consider the impact relations factor among the ESG variables. The second limitation stems from the evaluation of the field experts by linguistic variables. More quantitative data on firm-specific and country-specific variables should be used to achieve more refined results on the impact relations of ESG Index variables.

CRediT authorship contribution statement

Umit Hacioglu: Writing – original draft, Investigation, Modeling, Validation. **Hasan Dincer:** Writing – review & editing, Modeling, Analysis. **Mustafa Kemal Yilmaz:** Conceptualization, Supervision, Writing – review & editing. **Serhat Yüksel:** Methodology, Modeling, Writing – review & editing. **Mariama Sonko:** Conceptualization, Writing – review & editing, Validation. **Dursun Delen:** Writing – review & editing, Validation, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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All authors have read and agreed to the published version of the manuscript.

Appendix. Summary of the literature review in emerging markets

See Table A.1.

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