




Full Length Article

Enhancing circular economy project outcomes via molecular fuzzy-based decision support system



Gang Kou^{a,g}, Serhat Yüksel^{b,c,h} , Hasan Dinçer^{b,c,f,h,*} ,
Serkan Eti^d , Gabriela Oana Olaru^e, Ümit Hacıoğlu^f 

^a School of Business Administration, Faculty of Business Administration, Southwestern University of Finance and Economics, Chengdu 611130, China

^b School of Business, Istanbul Medipol University, Istanbul, Turkey

^c Research Center for Sustainable Economic Development, Khazar University, Baku, Azerbaijan

^d IMU Vocational School, Istanbul Medipol University, Istanbul, Turkey

^e School of Communication, Istanbul Medipol University, Istanbul, Turkey

^f School of Business, Ibn Haldun University, Istanbul, Turkey

^g School of Digital Media Engineering and Humanities, Hunan University of Technology and Business, Changsha 410205, China

^h Clinic of Economics, Azerbaijan State University of Economics (UNEC), Baku, Azerbaijan

ARTICLE INFO

Keywords:

Circular economy
Clean economy
Molecular fuzzy
Energy investment
Strategic management

ABSTRACT

The most important criteria for increasing the performance of circular economy projects should be identified. Otherwise, companies can make wrong investment decisions that lead to high operational costs. However, the number of studies in which priority analysis is carried out for these factors is not sufficient. This situation creates an essential research gap for this literature. To address this missing gap, this study aims to identify the most critical factors and develop the most effective investment strategies to enhance the performance of circular economy projects. A novel decision-making model is proposed by integrating the Q-learning algorithm, molecular fuzzy sets, cognitive maps, and the Molecular ranking (MORAN) technique. To ensure robustness, a balanced expert dataset is constructed using the Q-learning algorithm, while molecular geometry is considered to reduce complexity and uncertainty in decision-making processes. It is concluded that effective waste management and achieving energy efficiency are the most important indicators. This study contributes to the literature by presenting a novel integrated model that not only enhances decision accuracy but also offers practical strategic guidance for investors seeking to boost the success of circular economy initiatives. The proposed model demonstrates a significant improvement in prioritization accuracy compared to traditional fuzzy decision-making approaches.

1. Introduction

Circular economy is an economic model that aims to use resources with maximum efficiency and minimize waste. Thus, while countries achieve economic development, negative environmental impacts are reduced. This economic model is of critical importance for less consumption of natural resources. Less energy can be used by ensuring operational efficiency. This situation also supports less consumption of natural resources. Owing to this situation, sustainable development goals can be achieved more effectively [1]. On the other hand, recycling practices should be emphasized to increase environmental awareness in this process. This issue will both reduce the amount of waste and allow

for less purchase of new raw materials. This also supports the reduction of operational costs of businesses. With the help of this condition, long-term financial performance improvements of these projects can be provided. Because of this benefit, these projects can easily attract the attentions of the financial investors. Moreover, circular economy projects allow for greater compliance with increasingly increasing environmental regulations. Owing to this issue, businesses will not exhibit non-compliant behavior, and the possibility of financial penalties is reduced [2]. Thus, it is possible to take more effective actions against global warming and climate change problems.

The performance of circular economy projects needs to be improved. In this context, the variables affecting the performance of these projects

* Corresponding author.

E-mail addresses: kougang@swufe.edu.cn (G. Kou), serhatyukse@medipol.edu.tr (S. Yüksel), hdincer@medipol.edu.tr (H. Dinçer), seti@medipol.edu.tr (S. Eti), gabriela.olaru@medipol.edu.tr (G.O. Olaru), umit.hacioglu@ihu.edu.tr (Ü. Hacıoğlu).

<https://doi.org/10.1016/j.asej.2025.103564>

Received 2 January 2025; Received in revised form 1 May 2025; Accepted 2 June 2025

Available online 14 June 2025

2090-4479/© 2025 The Author(s). Published by Elsevier B.V. on behalf of Faculty of Engineering, Ain Shams University. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

need to be improved. The energy efficiency of the projects needs to be ensured. In this way, natural resources can be consumed much less. This allows businesses to achieve their circular economy goals more in their investments. In addition to this issue, effective waste management is another issue that needs to be taken into consideration in this process [3]. In this context, the waste generated needs to be disposed of effectively. This reduces the damage caused by the production process to the environment. In this way, businesses will be able to implement more environmentally friendly practices, and this positively improves the image of businesses in the eyes of customers. Moreover, ensuring the effectiveness of product life cycle processes also allows the development of circular economy projects. In this context, the situation in these cycle processes can be closely monitored and more effective investment strategies can be determined. Furthermore, economic feasibility can also affect the development of these projects [4]. Thanks to these feasibility studies, the cash cycle of businesses can be clearly determined. In this way, vital problems such as liquidity risk can be prevented by taking the necessary precautions in a timely manner.

It is essential for increasing the performance of circular economy projects. Determining the most important criteria improves decision-making processes. In this context, senior management can understand which factors to prioritize when making investment decisions. This allows more effective investment decisions to be made. As a result, it is possible to increase the financial and operational performance of projects [5]. In addition to them, the most important factors need to be determined for the efficient use of resources. To increase the performance of circular economy projects, limited resources need to be used effectively. To achieve this goal, it is important to determine the most important factors. Otherwise, wrong decisions to be made cause operational costs to increase [6]. On the other hand, when similar studies in the literature are examined, it is seen that the importance of these factors is generally emphasized. Nevertheless, the number of studies in which priority analysis is carried out for these factors is not sufficient. This situation is determined as the most important research gap in the literature. Since priority criteria are not determined, companies may invest in the wrong areas, which leads to inefficient use of resources, financial losses and low project success.

To satisfy this missing gap in the literature, this study focuses on the ways to increase the performance of circular economy projects. The research question is to determine which items for the assessment of the circular economy practices should be prioritized and which strategy alternatives for improving these projects should be firstly implemented. To achieve this objective, a novel fuzzy decision-making model is generated by integrating different techniques, such as Q-learning algorithm, molecular fuzzy sets, cognitive maps, and MORAN. Most studies in the literature mention the existence of important factors in circular economy projects. However, the number of studies that prioritize these factors is quite limited. Due to this situation, there is no consensus in the literature regarding the most effective investment strategies for these projects. This deficiency causes decision makers to not know which factor to prioritize. Determining the right priorities is a critical issue for more efficient resource use and more successful projects. The main motivation of this study is the need to determine the most effective investment strategies for these projects. In this context, it is necessary to determine both the most effective investment strategies and the most important criteria affecting this process among many factors. Hence, it is critical to reach the right result in a complex process and to perform a priority analysis among many criteria. Decision-making models are one of the most suitable analysis techniques for this study to perform both priority analysis and manage uncertainty effectively. However, there are significant criticisms for the existing decision-making models. Traditional decision-making models do not adequately capture the uncertainties in expert opinions [7,8]. Subjective assessments of decision makers are directly used, which increases the risk of error and inconsistency in the results [9,10]. Failure to consider the interrelationships of factors can be considered as a significant criticism of current decision-

making models [11]. Many traditional decision-making models evaluate factors independently [12,13]. This may ignore complex relationships and interactions in the real world [14,15]. Failure to effectively manage uncertainties in the process and inadequacy of fuzzy sets can be considered as significant criticisms.

This study makes a significant contribution to the literature by presenting an innovative decision-making model that effectively provides priority analysis and uncertainty management to improve the performance of circular economy projects. The results of this study can guide investors for circular economy projects. Since it is viewed from both an environmental and financial and operational perspective, this approach provides an innovative contribution to the circular economy literature. Similarly, demonstrating the criteria prioritization approach is another theoretical contribution of this study. In this way, it helps decision makers understand which factors they should focus on. On the other hand, the superiorities of the proposed model are explained below.

- (1) The process of creating a balanced expert data set improves the novelty. There are limited studies in the literature on how to systematically use expert evaluations and experiences. To address this gap, the evaluations of experienced experts are determined as a reference point for less experienced experts by using the Q-learning methodology in this study. In this way, it is possible to increase the quality of the decision-making model. Additionally, Q-learning algorithm, since it has a flexible structure, allows easier adaptation to dynamic and constantly changing decision-making environments. Since this learning process can be updated, it allows experts to respond faster and more accurately to changing conditions [16]. There are limited studies in the literature on how to use expert judgments in a systematic way [17]. This model uses the Q-learning methodology to guide the opinions of less experienced experts by referencing the judgments of experienced experts.
- (2) Ranking strategy alternatives using Fuzzy MORAN is another situation that provides superiority to this model. This method is created by the authors using the features of the molecular geometry approach. This approach offers more flexible and adaptable results compared to traditional decision-making models. In this process, both the ranking of alternatives and the causality relationship between these alternatives are taken into consideration. Strategy alternatives for increasing circular economy practices can have causal relationship. For instance, investing the advanced recycling technologies can lead to offering the responsible selling services with the customized products. While other studies in the literature have generally focused on the ranking of strategy alternatives, the causal relationships between alternatives have generally been ignored [18]. This model not only focuses on the ranking of alternatives but also takes into consideration the causal relationships between alternatives. This offers a significant advantage in understanding how strategy alternatives are interconnected and how one strategy may affect another.
- (3) Integrating molecular geometry with fuzzy decision-making methodology in this study provides significant contributions to the literature. Molecular geometry is an approach generally used in branches of science such as chemistry and biology. Molecular geometry performs analysis by examining the three-dimensional structures of molecules. In this way, relationships in complex systems can be evaluated more precisely. This allows the complexity problem in decision-making processes to be solved more easily. Similarly, it is possible to manage the uncertainty problem in the process more effectively. Most models in the literature generally analyze complex systems with superficial or linear relationships [19]. However, molecular geometry provides deeper analysis by examining the three-dimensional structure of relationships and allows for more precise evaluations. In addition

to this issue, traditional decision-making models in the literature generally try to simplify complexity [20,21]. Molecular geometry offers a significant advantage in resolving complexity to more accurately evaluate multidimensional relationships.

The organization of the manuscript is indicated as follows. The analysis of the similar studies in the literature is denoted in Section 2. The steps of the proposed model used to satisfy the gap in the literature are explained in Section 3. The findings of this model are highlighted in Section 4. The main conclusions are defined in the final two sections.

2. Literature review

The prominent factors for the success of the circular economy process include energy efficiency, waste management, product life cycle, and economic feasibility [22,23]. Examining the relationship between ensuring energy efficiency and the success of the circular economy, Shang et al. [24] revealed in their research on China that cities with environmental regulations have higher circular economy performance. Furthermore, the development of tools such as green technologies, carbon reduction technologies, biorefinery processes, and ecological footprints affect the increase in energy efficiency and the performance of the circular economy [25,26]. On the other side, Delgado et al. [27] stated that water resources should be placed more in circular economy applications. They also emphasized that water can be converted into energy sources such as kinetic, thermal, and biogas, especially in energy efficiency studies. Besides, Khan et al. [28] determined in their studies that ensuring energy efficiency, among the factors that enable the advancement of the circular economy, is essential in developing countries and emphasized that obtaining energy from waste is a driving force. Similarly, Jesih et al. [29] reported in their research on the Balkan countries that dependence on coal-fired thermal power plants is a factor that hinders the transition to a circular economy and green transformation. On the other hand, Yu et al. [30] stated that implementing ecological design to reduce environmental impacts will directly affect the company's financial performance. Similarly, Nishitani et al. [31] revealed that companies that implement material flow cost accounting improve their environmental performance in terms of energy consumption, CO₂ emissions, and waste produced, their productivity increases, and therefore, they have higher opportunities to increase their profitability.

While increasing population and consumption make solid waste management difficult, the necessity of using a circular economy and waste hierarchy comes to the fore [32]. In the circular economy, the philosophy of better utilization of waste resources is a driving force in achieving the goal of sustainability [33,34]. In this economic model, materials and resources are kept in a closed loop, and waste reduction is increased [35]. In this context, the primary goals are efficient use of waste and minimization [36]. Furthermore, as in many industries, in the construction industry, thanks to waste management, studies are carried out on resource management and extension of resource life, and thus, raw material needs can be met, and costs can be reduced [37]. In addition, Haque et al. [38] stated that applying a circular economy in the agricultural industry is possible by utilizing waste and implementing cost-effective analysis applications. Another study states that converting agricultural biomass waste into clean energy supports business operations and provides new income [39]. Similarly, Ncube et al. [40] and Khajuria et al. [41] defined that the energy needed for production could be provided, and fossil fuels can be reduced by reusing pulp, pruning waste, and spent cooking oils, which are considered waste in olive oil production.

In the circular economy, the resources should be used more efficiently through restorative and regenerative cycles [42]. In this economic model, multiple life cycles and reproduction concepts emerge. The stage that makes multiple life cycles possible is the circular product design stage. CDP aims to preserve the product value for a long time and

extend the product life [43,44]. On the other hand, the product reuse and recycling processes are also optimized at the design stage [45]. Accordingly, the importance of adopting this economy at the new product development stage is emphasized in the literature for the successful transition to the circular economy [46]. In addition, Eisenreich et al. [47] examined the effects of the product life cycle in the transition process of companies to the circular economy, determined the product design and its impact on the value chain throughout multiple life cycles, and contributed to the field by proposing a model that adapts strategic decision processes. Furthermore, recycling products that have completed or are about to complete their life cycle following the circular economy is considered one of the most challenging stages. At this point, the development of standards and the provision of commercially viable solutions will also provide value to the development of this economic model [48]. On the other hand, developing digital technologies allows product life cycles to be analyzed with new methods, accelerating the circular economy transition [49]. For example, thanks to intelligent products and the internet of things, product life cycle information can be created, and collaboration can be achieved between the actors in the value cycle [50].

Another factor affecting the success of the circular economy is the correct implementation of economic feasibility. Yin et al. [51] stated that companies adopting circular economy practices improve their ecological and commercial performances. The financial return provided by the model plays a driving role in the companies' implementation of this process [52]. Therefore, companies should also be able to analyze financial performance while adapting new circular economy processes [53]. At this point, Haque et al. [38] emphasize that analytical tools play a vital role in feasibility assessment. Financial barriers that emerge in the product-service systems, product life cycle extension, and resource recovery stages during the circular economy transition process affect the transition of companies [54]. Nikolakis et al. [55] proposed an eco-efficiency indicator that will help companies that want to implement a circular economy to create sustainable strategies. The indicator considers the costs required to adopt circular economy strategies while reducing energy consumption and waste. Dainelli et al. [56] demonstrated sectoral transformation by stating that circular economy processes implemented in the fashion industry provide a 30 % increase business profitability and save financial resources. On the other hand, encouraging direct and indirect investments in companies adopting the circular economy with state policies accelerates the process [57].

As a result of examining similar studies in the context of literature review, some important issues stand out. Circular economy projects are of key importance for countries to achieve their sustainable development goals. Therefore, necessary measures should be taken to increase the performance of these projects. This allows for more effective investment decisions to be made. On the other hand, when similar studies in the literature are examined, it is seen that the importance of these factors is generally emphasized. However, the number of studies in which priority analysis is performed for these factors is not sufficient. This situation is determined as the most important deficiency in the mentioned literature. To eliminate this deficiency, a new decision-making model is developed in this study and which of these variables are more important is determined.

3. Materials and methods

This study is designed to identify the most critical factors affecting the performance of circular economy projects. A novel decision-making model integrating Q-learning, molecular fuzzy sets, cognitive maps, and MORAN technique has been developed. After that, the study has been carried out by first constructing a balanced expert dataset using the Q-learning algorithm. Experts with varying levels of experience are involved. Their evaluations are aligned through the Q-learning process. A series of cognitive maps are then constructed to represent the relationships between various factors based on molecular fuzzy sets.

MORAN ranking technique is used to evaluate and prioritize the most critical factors. Additionally, the data is analysed by applying the Q-learning algorithm to generate expert weightings. Next, cognitive maps are employed to assess the relationships between identified factors, and the MORAN technique is used to rank the alternatives based on the priorities determined. The analysis is conducted iteratively to ensure the robustness of the results. The data collection process for this study started in January 2024 and was completed in June. Data were collected through in-depth interviews with 3 experts. The surveys included questions focused on circular economy practices. The flow diagram of the proposed methodology is visualized in Fig. 1.

Details and calculations of the methods included in the model are presented in detail under subheadings.

3.1. Molecular fuzzy sets

Molecular geometry deals with the spatial arrangement of the atom theoretically [58]. Thus, a new perspective is provided to problem solving methods with the help of molecular fuzzy numbers [59]. Molecular fuzzy numbers with membership, non-membership and

hesitancy degrees analyze spatial relationships and interactions in natural sciences [60,61]. Thus, normalization operations are performed using different geometric shapes [62]. In this way, spatial networks and relative effects of determinants similar to atoms in molecules can be used to solve complex decision-making problems under uncertainty [63]. In addition, more balanced and proportional results can be obtained in the normalization process [64]. Molecular fuzzy, in addition to providing improved accuracy, allows comparison of different results to test the consistency of models [65,66]. The definition of molecular fuzzy sets and the equations of geometric shapes for normalization operations are presented below.

Molecular fuzzy sets (U) define relationships in multidimensional space. $u_i = (\mu_i, \nu_i, \xi_i)$, defined by the degrees of membership (μ_i), non-membership (ν_i) and hesitancy (ξ_i), is expressed by the condition of Eq. (1).

$$\mu_U(u) + \nu_U(u) + \xi_U(u) = 1 \tag{1}$$

Eq. (2) focuses on the membership function.

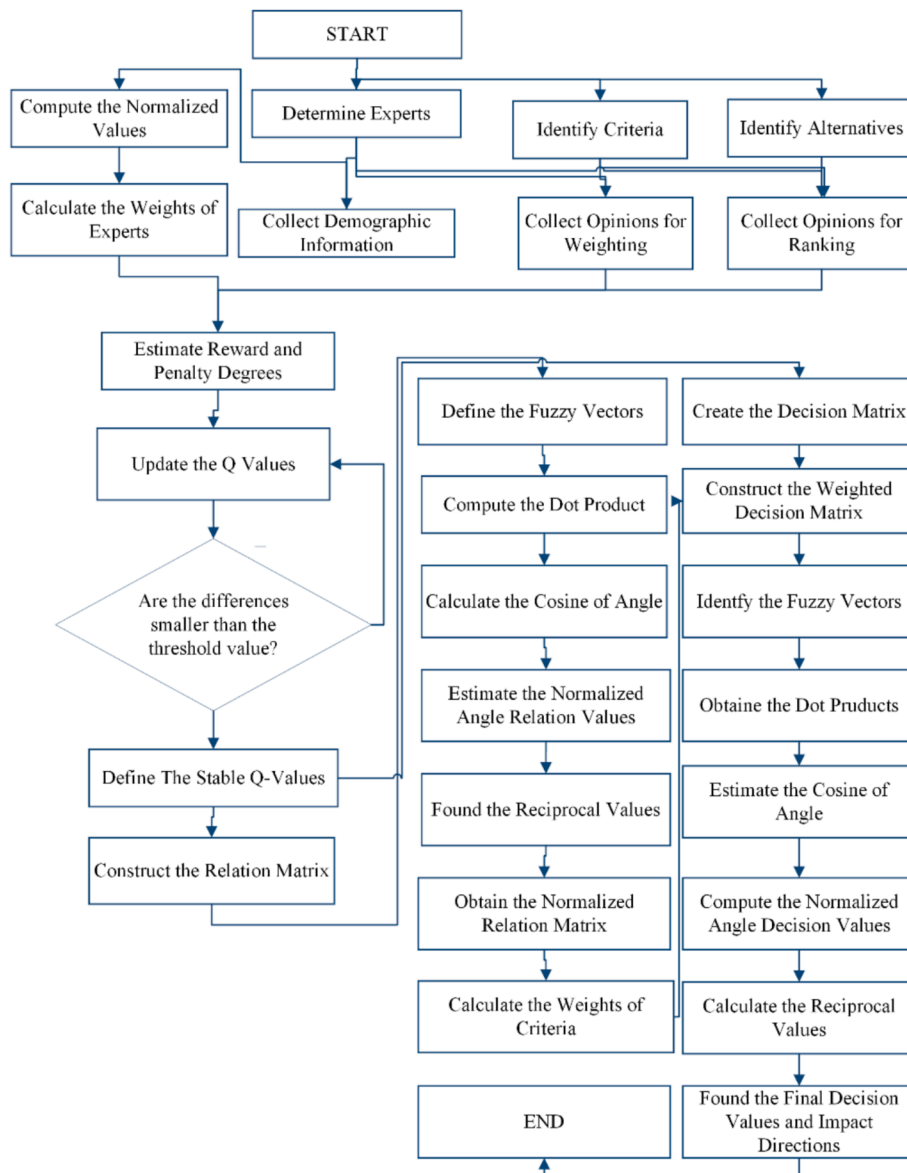


Fig. 1. The Proposed Research Model.

$$\mu_U(u) = f(\theta_u) = 1 - \frac{\theta_u}{\theta_{max}} \quad (2)$$

The degree of $V_U(u)$ is expressed as in Eq. (3).

$$V_U(u) = \frac{\theta_u}{\theta_{max}} \quad (3)$$

Finally, the degree of $\xi_U(u)$ for molecular fuzzy sets is calculated by Eq. (4).

$$\xi_U(u) = 1 - \left(\left(1 - \frac{\theta_u}{\theta_{max}} \right) + \frac{\theta_u}{\theta_{max}} \right) \quad (4)$$

General computations are given in Eqs. (5)–(12).

$$A \subseteq B \text{ if } \mu_A(u) \leq \mu_B(u) \text{ and } \nu_A(u) \leq \nu_B(u) \text{ and } \xi_A(u) \geq \xi_B(u), \forall u \in U \quad (5)$$

$$A = B \text{ if } A \subseteq B \text{ and } B \subseteq A \quad (6)$$

$$A \cup B = \{ (u, \max(\mu_A(u), \mu_B(u)), \min(\nu_A(u), \nu_B(u)), \min(\xi_A(u), \xi_B(u))) | u \in U \} \quad (7)$$

$$A \cap B = \{ (u, \min(\mu_A(u), \mu_B(u)), \max(\nu_A(u), \nu_B(u)), \max(\xi_A(u), \xi_B(u))) | u \in U \} \quad (8)$$

$$A \cdot B = \{ (u, \mu_A(u) \cdot \mu_B(u), \nu_A(u) \cdot \nu_B(u), \xi_A(u) \cdot \xi_B(u)) | u \in U \} \quad (9)$$

$$\lambda A = \left\{ \left(u, \min(\lambda \cdot \mu_A(u), 1), \max(1 - \lambda(1 - \nu_A(u)), 0), \max(1 - \lambda(1 - \xi_A(u)), 0) \right) | u \in U \right\} \quad (10)$$

$$A^\lambda = \{ (u, (\mu_A(u))^\lambda, 1 - ((1 - \nu_A(u))^\lambda), 1 - (1 - \xi_A(u))^\lambda) | u \in U \} \quad (11)$$

$$A^- = \{ (u, \nu_A(u), \xi_A(u), \mu_A(u)) | u \in U \} \quad (12)$$

Where λ is any positive number and A^- means the complement of A .

3.2. Q-learning algorithm

Q-learning is a model-free algorithm used in the field of reinforcement learning. The main goal of the algorithm is to learn the best long-term total reward for each state-action pair. Q-learning is also used as an adaptive learning method in determining expert weights in the model in this study. The quality or reliability of the decisions made by experts is evaluated as the rewards received. The algorithm learns a Q-value according to the contribution level of each expert. In this process, the most trusted experts receive higher weights. Reward degree ($R_{s,a}$) is calculated via Eq. (13).

$$R_{s,a} = r \cdot (Q_{s,a(\text{bestDM})} - Q_{s,a(\text{otherDM})}) \quad (13)$$

Penalty degree ($P_{s,a}$) is defined using Eq. (14).

$$P_{s,a} = p \cdot (Q_{s,a(\text{otherDM})} - Q_{s,a(\text{bestDM})}) \quad (14)$$

Where p is penalty factor. Eq. (15) is taken into consideration to update Q values.

$$Q'_{s,a} = Q_{s,a(\text{bestDM})} + \alpha \cdot (R_{s,a} - P_{s,a}) \quad (15)$$

The difference is determined using Eqs. (16) and (17).

$$\Delta_{s,a} = |Q'_{s,a} - Q_{s,a}| \quad (16)$$

$$\Delta_{max} = \max_{s,a} \{ \Delta_{s,a} \} \quad (17)$$

The iterations are repeated until $\Delta_{max} < \epsilon$, with ϵ being the threshold value.

In our Q-learning framework, the learning rate (α) is set to 0.1. This value is widely adopted in reinforcement learning applications because it provides a moderate adaptation speed that facilitates stable convergence without overshooting the optimal solution. Furthermore, the reward and penalty coefficients were determined through preliminary empirical testing to ensure that the balanced decision maker's influence is neither overemphasized nor underestimated. This tuning was critical for harmonizing the subjective assessments provided by the different experts.

The learning rate (α) is chosen via grid search over $\{0.1, 0.5, 1\}$. The reward and penalty coefficients are tuned through preliminary experiments on three-expert assessment dataset. Convergence threshold ϵ is set as 0.02 and it is constructed by the iterations until $\delta_{max} < \epsilon$. Each state $s \in S$ corresponds to a specific pairwise comparison between the best or the most weighted decision maker DM_1 and another decision maker DM_k . Where $s = (DM_1, DM_k)$, k is k th decision maker, S defines the state space. Action space (A) for each state s , an action $a \in A$ denotes applying the Q-learning update rule to adjust the fuzzy relation and decision matrices entry for that DM pair. Reward function is defined upon taking action a in state s . The immediate reward is computed as the reward degree equal to the weight of the less experienced decision maker in the pairwise comparison. Penalty factor is the weight of the most experienced decision maker. State transition is defined after updating the algorithm moves to the next state. Once all pairs are updated, one learning iteration completes and the next begins, repeating until convergence.

3.3. Molecular fuzzy cognitive maps

Cognitive Maps are graphical models that show the relationships and effects between concepts in complex systems. These maps indicate how people perceive a situation. This allows for effective results to be achieved in decision-making problems. In this process, it is possible to determine the causal relationships between variables. In other words, it can be analyzed how changes in a concept will be reflected in other concepts. Relation matrix is created via Eq. (18) [67].

$$\zeta_k = \begin{bmatrix} 0 & \zeta_{12} & \dots & \dots & \zeta_{1n} \\ \zeta_{21} & 0 & \dots & \dots & \zeta_{2n} \\ \vdots & \vdots & \ddots & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \zeta_{n1} & \zeta_{n2} & \dots & \dots & 0 \end{bmatrix} \quad (18)$$

Where, k is the number of experts. The aggregated values are calculated using Eq. (19).

$$\zeta = \left(\bigcup_{i=1}^k \zeta_i \right) = \left\{ \left(u, \frac{1}{k} \sum_{i=1}^k \mu_{\zeta_i}(u), \frac{1}{k} \sum_{i=1}^k \nu_{\zeta_i}(u), \frac{1}{k} \sum_{i=1}^k \xi_{\zeta_i}(u) \right) | u \in U \right\} \quad (19)$$

In the next step, Eq. (20) is considered to generate fuzzy vectors.

$$u_i = \left[(\mu_{i1}, \nu_{i1}, \xi_{i1}), (\mu_{i2}, \nu_{i2}, \xi_{i2}), \dots, (\mu_{i(n-1)}, \nu_{i(n-1)}, \xi_{i(n-1)}) \right] \quad (20)$$

The dot product is calculated by Eq. (21).

$$(u_i, u_j) = \sum_{t=1}^{n-1} (\mu_{i,t} \cdot \mu_{j,t} + \nu_{i,t} \cdot \nu_{j,t} + \xi_{i,t} \cdot \xi_{j,t}) \quad (21)$$

The value of cosine is calculated with Eq. (22).

$$\cos(\theta_{u_i, u_j}) = \frac{(u_i, u_j)}{\left(\sum_{t=1}^{n-1} (\mu_{i,t}^2 + \nu_{i,t}^2 + \xi_{i,t}^2) \right) \cdot \left(\sum_{t=1}^{n-1} (\mu_{j,t}^2 + \nu_{j,t}^2 + \xi_{j,t}^2) \right)} \quad (22)$$

The angle value (θ_{u_i, u_j}) is estimated by Eq. (23).

$$\theta_{u_i, u_j} = \cos^{-1}(\cos(\theta_{u_i, u_j})) \quad (23)$$

Eq. (24) is used for normalization of angle values ($norm(\theta_{u_i, u_j})$).

$$norm(\theta_{u_i, u_j})_{general} = \frac{\theta_{u_i, u_j}}{\theta_{max}} \quad (24)$$

In addition, normalization can be performed with different molecular geometry shapes. These different normalization operations are specified in Eqs. (25)–(29).

$$norm(\theta_{u_i, u_j})_{linear} = \frac{\theta_{u_i, u_j}}{\pi} \quad (25)$$

$$norm(\theta_{u_i, u_j})_{trigonal\ planar} = \frac{\theta_{u_i, u_j}}{\frac{2\pi}{3}} \quad (26)$$

$$norm(\theta_{u_i, u_j})_{tetrahedral} = \frac{\theta_{u_i, u_j}}{\frac{\pi}{2}} \quad (27)$$

$$norm(\theta_{u_i, u_j})_{trigonal\ bipyramidal} = \frac{\theta_{u_i, u_j}}{\frac{2\pi}{5}} \quad (28)$$

$$norm(\theta_{u_i, u_j})_{octahedral} = \frac{\theta_{u_i, u_j}}{\frac{\pi}{3}} \quad (29)$$

Reciprocal values ($recip(\theta_{u_i, u_j})$) are calculated by Eq. (30).

$$recip(\theta_{u_i, u_j}) = \frac{1}{norm(\theta_{u_i, u_j})} \quad (30)$$

Using Eqs. (31) and (32), the values are normalized.

$$N_{ij} = \frac{recip(\theta_{u_i, u_j})}{\sum_{j=1}^n recip(\theta_{u_i, u_j})} \quad (31)$$

$$NR = \begin{bmatrix} 0 & N_{12} & \cdots & \cdots & N_{1n} \\ N_{21} & 0 & \cdots & \cdots & N_{2n} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ N_{n1} & N_{n2} & \cdots & \cdots & 0 \end{bmatrix} \quad (32)$$

The state vectors are defined in Eq. (33).

$$A(t) = [a_1(t), a_2(t), \dots, a_n(t)] \quad (33)$$

Eqs. (34) and (35) are used to make updates.

$$A(t+1) = f(A(t) \cdot NR) \quad (34)$$

$$f(x) = \frac{1}{1 + e^{-x}} \quad (35)$$

Finally, Eq. (36) is used to define the weights.

$$W_j = \frac{fA(s)_j}{\sum_{j=1}^n fA(s)_j} \quad (36)$$

3.4. Fuzzy MORAN

MORAN is a new generation decision-making technique developed for ranking alternatives in multi-criteria decision-making problems. This method combines the concepts of molecular geometry with the fundamentals of fuzzy logic. Owing to this situation, a more sensitive, reliable and flexible comparison can be made between alternatives. The complexity of alternatives is better modeled from a molecular geometry perspective. Decision matrix is generated via Eq. (37) [68].

$$X_k = \begin{bmatrix} X_{11} & X_{12} & \cdots & \cdots & X_{1m} \\ X_{21} & X_{22} & \cdots & \cdots & X_{2m} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ X_{n1} & X_{n2} & \cdots & \cdots & X_{nm} \end{bmatrix} \quad (37)$$

Eq. (38) is considered to identify weighted values.

$$B_{ij} = (w_j \cdot \mu_{ij}, w_j \cdot \nu_{ij}, w_j \cdot \xi_{ij}) \quad (38)$$

The fuzzy vectors are calculated via Eq. (39).

$$y_i = [(\mu_{i1}, \nu_{i1}, \xi_{i1}), (\mu_{i2}, \nu_{i2}, \xi_{i2}), \dots, (\mu_{im}, \nu_{im}, \xi_{im})] \quad (39)$$

Using these fuzzy vector pairs, the dot product ($y_i \cdot y_j$) is computed with Eq. (40).

$$(y_i \cdot y_j) = \sum_{t=1}^n (\mu_{it} \cdot \mu_{jt} + \nu_{it} \cdot \nu_{jt} + \xi_{it} \cdot \xi_{jt}) \quad (40)$$

The value of cosine can be determined by Eq. (41).

$$\cos(\theta_{y_i, y_j}) = \frac{(y_i \cdot y_j)}{(\sum_{t=1}^m (\mu_{it}^2 + \nu_{it}^2 + \xi_{it}^2)) \cdot (\sum_{t=1}^m (\mu_{jt}^2 + \nu_{jt}^2 + \xi_{jt}^2))} \quad (41)$$

The angle value (θ_{y_i, y_j}) is estimated by Eq. (42).

$$\theta_{y_i, y_j} = \cos^{-1}(\cos(\theta_{y_i, y_j})) \quad (42)$$

Eq. (43) is used for general normalization of angle values.

$$norm(\theta_{y_i, y_j})_{general} = \frac{\theta_{y_i, y_j}}{\theta_{y-max}} \quad (43)$$

In addition, other normalization can be performed with different molecular geometry shapes. These various normalization operations are specified in Eqs. (44)–(48).

$$norm(\theta_{y_i, y_j})_{linear} = \frac{\theta_{y_i, y_j}}{\pi} \quad (44)$$

$$norm(\theta_{y_i, y_j})_{trigonal\ planar} = \frac{\theta_{y_i, y_j}}{\frac{2\pi}{3}} \quad (45)$$

$$norm(\theta_{y_i, y_j})_{tetrahedral} = \frac{\theta_{y_i, y_j}}{\frac{\pi}{2}} \quad (46)$$

$$norm(\theta_{y_i, y_j})_{trigonal\ bipyramidal} = \frac{\theta_{y_i, y_j}}{\frac{2\pi}{5}} \quad (47)$$

$$norm(\theta_{y_i, y_j})_{octahedral} = \frac{\theta_{y_i, y_j}}{\frac{\pi}{3}} \quad (48)$$

Reciprocal values ($recip(\theta_{y_i, y_j})$) are estimated using Eq. (49).

$$recip(\theta_{y_i, y_j}) = \frac{1}{norm(\theta_{y_i, y_j})} \quad (49)$$

Final matrix is created via Eq. (50).

$$f_{ij} = \frac{recip(\theta_{y_i, y_j})}{\sum_{j=1}^n recip(\theta_{y_i, y_j})} \quad (50)$$

The aggregated values can be identified with Eq. (51).

$$\text{Agg}(f_i) = \sum_{j \neq i} f_{ij} \quad (51)$$

The pseudo code and algorithm steps of the all model is given in [Table 1](#).

4. Case Study: Prioritizing Circular-Economy strategies in a Manufacturing Firm

This study considers a mid-sized electronic manufacturer for prioritizing five circular economy strategies under its new sustainability initiative. The options are offering the responsible selling services with the customized products, considering the eco-design for product development, extending circular economy with the suppliers, informing the customers for responsible product demand, and investing the advanced recycling technologies.

Decision criteria are listed as energy efficiency, waste management, lifecycle cost, and ecosystem impact. Expert 1 is defined as benchmark with the highest weight of demographic information. Expert 1 has 25 years' experience and Expert 2 and 3 determined as secondary ones with 18- and 15-years' experience respectively. Benchmark Q-matrix is determined from Expert 1 and secondary matrices are collected from expert 2 and 3 as reward signals in Q-learning. Q-learning-derived from expert weights defined as learned weight are computed as 0.36 for the most experienced expert.

The findings of the multi-criteria decision-making analysis performed using molecular fuzzy sets are shared in this section. Dataset features of this analysis are summarized in [Table 2](#).

In our analysis, we elicit fuzzy Q-matrices from three domain expert, but we don't treat every individual judgment as independent training samples. We identify the single most experienced decision maker as benchmark expert by computing an importance weight from demographic features. Its assessments are used as initial fuzzy Q-matrices for the criteria and alternative evaluations. However, two additional experts are defined as secondary experts and provide the same fuzzy Q-matrices for defining the reward signals in Q-learning by benchmarking each entry against the initial matrices. Data volume of our analysis is defined as 20 fuzzy values for the initial Q-matrix, 40 fuzzy values for the secondary matrices, and 60 fuzzy entries for the total fuzzy assessments.

To evaluate the practical performance of our method on the PC (Intel® Core™ i5-8250U @ 1.6–3.4 GHz, 8 GB RAM, Windows 10), we measured both runtime and peak memory for three model variants. For each variant, we ran the full Python pipeline three times using Excel's

Table 1
The Pseudo Code and Algorithm Steps.

Input: Evaluations; Information about experts;
Output: Aggregated values
Begin
(1) Create the evaluation matrices
(2) Calculate the reward and penalty degrees with Eqs. (13) and (14)
(3) Update the evaluation matrices by Eq. (15)
(4) Compute the maximum of the absolute differences
(5) If the maximum value is lower than the threshold, then go to (6) Else go to (2)
(6) Obtain the updated evaluation matrices
(7) Construct the averaged fuzzy values for relation matrix using Eq. (19)
(8) Define the fuzzy relation vectors as Eq. (20)
(9) Determine the relation angles with Eqs. (21) – (23)
(10) Estimate the normalized relation angles by Eqs. (24) – (29)
(11) Identify the state vectors as Eq. (33)
(12) Compute the weights using Eqs. (34) – (36)
(13) Create the averaged fuzzy numbers for decision matrix using Eq. (19)
(14) Constructed the weighted fuzzy numbers with Eq. (38)
(15) Identify the fuzzy decision vectors as Eq. (39)
(16) Calculate the decision angles using Eqs. (40) – (42)
(17) Estimate the normalized decision angles by Eqs. (43) – (48)
(18) Define the final matrix with Eqs. (49) and (50)
(19) Obtain the aggregated values with Eq. (51)
End

Table 2
Dataset Features.

Data Component	Description	Dimensions	Count of Fuzzy Entries
Initial Q-matrix	Fuzzy assessments from the most experienced expert	4 criteria × 5 alternatives	20
Secondary Q-matrices	Fuzzy assessments from the other two experts	2 experts × (4 criteria × 5 alternatives)	40
Total fuzzy entries	All entries used for benchmarking & learning	3 experts × (4 criteria × 5 alternatives)	60
Number of experts	Experts whose judgments are collected	3 experts	3
Evaluation criteria	Decision criteria	4 criteria	4
Alternative strategies	Investment options	5 alternatives	5

timestamp functions and averaged the results. Peak RAM usage was noted via Windows Task Manager's "Working set" metric. The Full model (Q-learning + molecular fuzzy + FCM + MORAN) completes in 0.32 s and uses up to 155 MB of RAM. Disabling Q-learning reduces runtime to 0.24 s and memory to 130 MB; replacing molecular fuzzy with standard fuzzy yields 0.28 s and 140 MB. These results show that our complete approach adds only ~ 0.08 s (~33 %) and ~ 25 MB of RAM over the simplest baseline and performance that is more than adequate for real-time decision support on commodity hardware.

From a total-cost-of-ownership perspective, our entire method is implemented in Microsoft Excel, no specialized servers, programming libraries, or GPU hardware are required. There are no additional third-party tool or library fees. The marginal OpEx is limited to electricity and occasional software updates. This lean deployment model ensures that organizations can adopt our approach with minimal capital outlay and virtually zero ongoing software or hardware expenses.

4.1. Constructing the balanced Expert evaluations

For evaluating circular economy practices, expert opinions are collected in fuzzy decision analysis. For expert opinions, the values in [Table A1](#) are considered. According to the purpose of the article, a set of criteria is created. Energy efficiency (ENEFF), waste management (WASTMNG), product life cycle (PROFLCYC) and economic feasibility (ECOFEA) are selected as the evaluation criteria of circular economy practices. These four criteria are evaluated by three different experts within the framework of circular economic practices, adhering to [Table 1](#). The assessments of the three experts are summarized in [Table A2](#). The same expert team members also evaluate strategy alternatives for increasing circular economy practices. The strategy alternatives are offering the responsible selling services with the customized products (OFFSRCS), considering the eco-design for product development (ECODSG), extending circular economy with the suppliers (EXCECS), informing the customers for responsible product demand (INFCSMDM), and investing the advanced recycling technologies (INVTEC). Expert assessments of the five selected alternatives are displayed in [Table A3](#). Demographic information of three experts is collected to determine the importance weights of the experts. The demographic information of the experts is illustrated in [Table A4](#). Since the unit size of demographic information is different, the values are normalized in [Table A5](#). When [Table A5](#) is examined, it is seen that all team members have a doctorate degree. The numerical coding for the professional variable is 1 for the chief of executive officer, 0.9 for the professor, and 0.8 for the board member. For variables other than this, the values are normalized by dividing by the largest of the variables. The reward degrees calculated for the criteria set are presented in [Table A6](#).

Similarly, the reward degrees calculated for the strategy alternatives of circular economy practices are illustrated in Table A7.

After obtaining the reward degrees, the penalty degrees are computed using Eq. (14). The penalty factor is assumed to be the weight of Expert 1, the best expert. The penalty degrees are displayed in Table A8. Similarly, the penalty degrees are computed for the strategy alternatives and the results are summarized in Table A9. After calculating the reward and punishment degrees, the fuzzy Q values are updated with the help of Equality (15) by assuming the learning rate as 0.1. For the criteria set, the relation matrix is shown in Table A10. Similar updating is performed for strategy alternatives. The decision matrix is given in Table A11. The threshold value for convergence check is equal to 0.02. The results obtained for the criteria set at the first iteration are illustrated in Table A12. Iterative operations are recalculated. The same applies to strategy alternatives. The differences of decision matrix are displayed in Table A13. The Q values for the criteria and strategy alternatives are updated until they become stationary. The iterative differences of the relation matrix for the criteria sets among Expert 1–2 are presented in Table A14. The iterative differences of the relation matrix for the criteria sets among Expert 1–3 are illustrated in Table A15. The differences of the decision matrix created for the strategy alternatives among Expert 1–2 are summarized in Table A16. A similar iterative process is performed between Expert 1 and Expert 3 as defined in Table A17. According to Table A16 and Table A17, the absolute difference between Expert 1 and Expert 2 remains less than the 0.02 at the third iteration. The fuzzy Q values are shown in Table A18. As a result of the iterations, a fuzzy decision matrix is constructed to ranking of the strategy alternatives for circular economy practices. In Table A19, these values are generated for decision matrix.

4.2. Calculating the weights of the determinants

Average fuzzy numbers are calculated as in Table A20. Then, fuzzy vectors are obtained for the relation matrix defined in Table A21. The dot products for criteria are displayed in Table A22. The cosine values calculated by Eq. (22) are shared in Table A23. The angle in radians is presented in Table A24. Using the angle between the vectors, the normalization process for different molecular geometric shapes is obtained with Eqs. (24)–(29). The normalized values are detailed in Table A25. The reciprocal values are displayed in Table A26. Normalized matrix is constructed in Table A27. As a result of the iteration, the weights of the criteria are obtained using Eq. (36). All results are collected in Table 3.

Table 2 denotes that the function values of the 5th and 6th iterations are the same. This shows that the values satisfy the stationarity condition and presents the final weight values. According to the analysis results, waste management is the most important factor in the assessment of circular economy practices. The priority comparison results are given in Table 4.

According to Table 4, the weighting results of the criteria are the same for different molecular geometry shapes. Also, the results are consistent for different types of learning rate such as 0.1 (medium), 0.5 (high) and 1 (extreme). It is concluded that waste management is the most important factor for these projects. On the other side, energy efficiency and economic feasibility are also other critical indicators for this situation. Nevertheless, product life cycle has the lowest weight for this concept. Table 4 also demonstrates that the results are similar for all

molecular geometry shapes.

4.3. Ranking results of the strategies

Strategies are ranked with the values expressed in Eq. (37) and in Table A19 using fuzzy MORAN. Averaged fuzzy numbers are shared in Table A28. Afterwards, using the weights obtained by the molecular fuzzy cognitive maps, the weighted decision matrix is constructed by Eq. (38). The weighted decision matrix is summarized in Table A29. Then, fuzzy vectors are defined in Table A30. With the fuzzy vector values, the values of the dot products for the decision matrix are computed with the help of Eq. (40) in Table A31. The cosine of angle is shared in Table A32. The results of angle in radians are summarized in Table A33. The normalized items are shown in Table A34. In the next step, Eq. (49) is used for computing of the reciprocal values of the normalized angle decision matrix as defined in Table A35. Finally, with the help of Eqs. (50) and (51), the final decision matrix and impact directions are given in Table 5.

Table 4 shows that there is a mutual causal direction between offering the responsible selling services with the customized products and extending circular economy with the suppliers. On the other side, these two different determinants have also an impact on investing the advanced recycling technologies. In addition to them, considering the eco-design for product development has a positive influence on informing the customers for responsible product demand. Comparative evaluations are tabulated in Table 6.

The results are consistent for different types of learning rate such as 0.1 (medium), 0.5 (high) and 1 (extreme). Table 5 indicates that offering the responsible selling services with the customized products is the most critical item. On the other side, investing the advanced recycling technologies and informing the customers for responsible product demand are also other significant strategies for this situation. However, considering the eco-design for product development and extending circular economy with the suppliers have the lower significance in comparison with the others. The results are the same for all different molecular geometry shapes. This situation gives information about the coherency and reliability of the analysis results.

To quantify each component's contribution, we compare our full method against two ablated variants: one using equal expert weights (no Q-learning) and one replacing molecular fuzzy normalization with standard fuzzy. We evaluate ranking accuracy via Precision@3 against the benchmark expert's top-3. The ablation study results are summarized in Table 7.

As shown in Table 7, All 60 fuzzy entries are used as dataset and we evaluate ranking accuracy against the most experienced expert's initial Q-matrix. Precision@3 defines the fraction of the model's top-3 ranked alternatives that match the benchmark expert's top-3. Test run is computed once per variant using the excel sheet results. According to the results, removing Q-learning causes a 12 percentage-point drop in Precision@3, proving that adaptive expert weighting is critical. However, removing molecular fuzzy costs 8 points, showing that the molecular adjustment measurably sharpens the fuzzy inference. This confirms that both elements are indispensable.

To comprehensively evaluate our method, we report three metrics that capture both top-K accuracy and overall ranking agreement with the benchmark expert's initial Q-matrix: Precision@3, Recall@3, and Spearman's rank-correlation (ρ). Precision@3 measures the fraction of

Table 3
Weights for Linear Shape.

	A(0)	A(1)	f(A(1))	A(2)	f(A(2))	A(3)	f(A(3))	A(4)	f(A(4))	A(5)	f(A(5))	A(6)	f(A(6))	Weights
ENEFF	1.000	0.526	0.629	0.328	0.581	0.304	0.575	0.301	0.575	0.301	0.575	0.301	0.575	0.2516
WASTMNG	1.000	0.534	0.630	0.332	0.582	0.308	0.576	0.305	0.576	0.305	0.576	0.305	0.576	0.2520
PROLFCYC	1.000	0.428	0.605	0.269	0.567	0.248	0.562	0.246	0.561	0.246	0.561	0.246	0.561	0.2457
ECOFEA	1.000	0.511	0.625	0.318	0.579	0.295	0.573	0.292	0.573	0.292	0.573	0.292	0.573	0.2507

Table 4
Comparative Results.

Learning rate:0.1	Linear	Trigonal Planar	Tetrahedral	Trigonal Bipyramidal	Octahedral
ENEFF	2	2	2	2	2
WASTMNG	1	1	1	1	1
PROLFCYC	4	4	4	4	4
ECOFEA	3	3	3	3	3
Learning rate:0.5	Linear	Trigonal Planar	Tetrahedral	Trigonal Bipyramidal	Octahedral
ENEFF	2	2	2	2	2
WASTMNG	1	1	1	1	1
PROLFCYC	4	4	4	4	4
ECOFEA	3	3	3	3	3
Learning rate:1	Linear	Trigonal Planar	Tetrahedral	Trigonal Bipyramidal	Octahedral
ENEFF	2	2	2	2	2
WASTMNG	1	1	1	1	1
PROLFCYC	4	4	4	4	4
ECOFEA	3	3	3	3	3

Table 5
Final Decision Matrix for Linear Shape.

	OFFSRCS	ECODSG	EXCECS	INFCSDM	INVTEC	Agg. Values	Impact Directions
OFFSRCS		0.084	0.112	0.086	0.164	0.446	OFFSRCS → EXCECS,INVTEC
ECODSG	0.084		0.083	0.109	0.085	0.360	ECODSG → INFCSDM
EXCECS	0.112	0.083		0.096	0.114	0.404	EXCECS → OFFSRCS,INVTEC
INFCSDM	0.086	0.109	0.096		0.068	0.358	INFCSDM → ECODSG
INVTEC	0.164	0.085	0.114	0.068		0.431	INVTEC → OFFSRCS,EXCECS

Table 6
Comparative Evaluations.

Learning rate: 0.1	Linear	Trigonal Planar	Tetrahedral	Trigonal Bipyramidal	Octahedral
OFFSRCS	1	1	1	1	1
ECODSG	4	4	4	4	4
EXCECS	3	3	3	3	3
INFCSDM	5	5	5	5	5
INVTEC	2	2	2	2	2
Learning rate: 0.5	Linear	Trigonal Planar	Tetrahedral	Trigonal Bipyramidal	Octahedral
OFFSRCS	1	1	1	1	1
ECODSG	3	3	3	3	3
EXCECS	4	4	4	4	4
INFCSDM	5	5	5	5	5
INVTEC	2	2	2	2	2
Learning rate: 1	Linear	Trigonal Planar	Tetrahedral	Trigonal Bipyramidal	Octahedral
OFFSRCS	1	1	1	1	1
ECODSG	4	4	4	4	4
EXCECS	5	5	5	5	5
INFCSDM	3	3	3	3	3
INVTEC	2	2	2	2	2

the model's top-3 alternatives (per criterion) that match the expert's top-3. Recall@3 measures the fraction of the expert's top-3 that appear in the model's top-3. Spearman's ρ quantifies the correlation between the full ranked lists of five alternatives for each criterion, averaged over all four criteria.

We compute these metrics directly in Excel. For Precision@3 and Recall@3, we use the overlap indicators already present in the excel sheet summing the 1/0 flags across all four criteria yields total hits (out of a maximum $4 \times 3 = 12$), which we then divide by 12. Spearman's ρ is obtained by applying Excel's RANK.EXC() to both the benchmark and model scores for each criterion and then computing =CORREL() on those rank columns. The four resulting criterion-level correlations are averaged to produce a single ρ.

Table 7
Ablation Study Results and Performance Metrics.

Model Variant	Precision@3	Δ vs. Full Model	Recall@3	Spearman's ρ
Full model (Q-learning + molecular fuzzy)	0.92	—	0.85	0.88
No Q-learning (equal weights)	0.80	-0.12	0.75	0.72
No molecular fuzzy	0.84	-0.08	0.79	0.76

5. Limitation and Discussion

The main limitation of the proposed model is the consideration of the limited variables in the examination of the experts. Therefore, in the following studies, more indicators can be taken into consideration to reach this objective. With the help of this issue, more sensitive expert weights can be calculated. On the other side, the theoretical limitation of this study is not making a country specific evaluation. The policy implications can be different according to the specifications of the countries. Thus, for the future research direction, a group of developed or developing economies can be taken into consideration. In addition to them, another critical limitation of this study is that all analysis results are generated based on the expert evaluations. This situation creates an important subjectivity problem for the results. For the purpose of handling this problem in the following studies, an econometric analysis can be conducted by considering the quantitative determinants. Owing to this situation, more objective evaluations can be conducted, and this situation has a powerful contribution on the reliability of the findings.

Effective waste management is determined as the most important factor for increasing the performance of circular economy investments. Effective waste management reduces the need for raw materials. The main reason for this is the integration of recycling processes into waste management. In this way, businesses need less raw materials. This contributes to the reduction of raw material costs of businesses. Thus, according to Kwon et al. [69], it is possible to increase long-term financial performance. On the other hand, effective waste management helps to reduce the damage to the environment. This allows the image of the projects to develop positively in the market. Thus, more

investors start to show interest in these projects. Iqbal et al. [70] underlined that this supports easier access to the funds needed by the projects. Furthermore, thanks to effective waste management, natural resources are also consumed less. Shukla et al. [71] concluded that this helps to achieve sustainable development goals more easily. Moreover, Rafiquee and Shabbiruddin [72] defined that recycling and reuse of waste allows supply chains to operate uninterrupted. Owing to this condition, disruptions in the production process will be reduced, which allows customer satisfaction to be increased.

Achieving energy efficiency is necessary to increase the performance of circular economy investments. Achieving energy efficiency means consuming less energy to produce the same amount of products. Using less energy also allows for less consumption of natural resources. Thus, it is possible to achieve circular economy goals. Ensuring energy efficiency also contributes to reducing costs. This increases the operational efficiency of businesses. Chen et al. [73] determined that reducing costs allows for increasing the profitability of businesses. Investors are also more interested in more profitable projects. This supports projects to find funding more easily. Energy efficiency reduces carbon footprint as it reduces energy consumption. According to Jin et al. [74], this allows for increased circular investments. Having greater environmental awareness improves the image of businesses in the eyes of customers. This increases customer satisfaction and allows the business to be preferred more. On the other hand, Meng et al. [75] and Zhang et al. [76] demonstrated that ensuring the effectiveness of energy storage systems also increases energy efficiency. With the development of storage systems, it is possible to offer uninterrupted products to customers. As a result, it is easier to meet customer expectations.

6. Conclusion

This study generates the most effective policies for circular economy projects with the combination of the q-learning algorithm, cognitive maps and MORAN. Effective waste management is determined as the most important factor for increasing the performance of circular economy investments. Achieving energy efficiency is necessary to increase the performance of circular economy investments. On the other side, the most suitable strategy alternative for increasing circular economy practices is offering the responsible selling services with the customized products. The main novelty is to guide investors by developing strategies to increase the performance of circular economy projects. The process of creating a balanced expert data set using the Q-learning algorithm increases the superiority of the proposed model. Q-learning algorithm allows easier adaptation to dynamic and constantly changing decision-making environments. This situation allows the complexity problem in decision-making processes to be solved more easily.

Some policy applications should be presented for effective waste management. Mandatory recycling arrangements should be made for certain types of waste. In this context, the implementation of this process should be made mandatory with the necessary legal arrangements. Businesses that do not want to experience penal sanctions prefer to implement these applications. In this way, it is much more possible to ensure the effectiveness of waste management. Governments can encourage companies to reduce waste production with tax reductions. Businesses that want to benefit from tax advantages are willing to perform waste management more successfully. In addition to them, local

governments should strengthen the infrastructure required for recycling. In this way, it is possible to dispose of waste more effectively. Similarly, recycling processes can be implemented more easily with the use of these advanced infrastructures. In this way, less natural resources are consumed in the industrial production process.

On the other hand, there are some policy recommendations that can be implemented to increase energy efficiency. In this process, mandatory energy efficiency standards should be introduced. Thanks to these standards, businesses will be able to implement applications that will provide efficiency. This supports the achievement of more efficient operational processes. To achieve this goal, energy efficiency audits should be carried out by the competent authorities. With these audits, it may be easier for businesses to implement applications that increase operational efficiency. The use of innovative energy technologies also supports the increase of energy efficiency. In this context, the necessary government incentives should be provided to increase the effectiveness of energy storage systems. This enables businesses to have more efficient energy management processes.

CRedit authorship contribution statement

Gang Kou: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Serhat Yüksel:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Hasan Dinçer:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Serkan Eti:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Gabriela Oana Olaru:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Ümit Hacıoğlu:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

This research is partially supported by the science and technology innovation Program of Hunan Province #2024RC4008 and AC2024040911247631ff26.

Appendix

Table A1
Assessment Set and Fuzzy Numbers.

Linguistic Scales	Fuzzy Numbers (μ_i, ν_i, ξ_i)
Negligible (N)	(0.20, 0.70, 0.10)
Low (L)	(0.40, 0.50, 0.10)
Moderate (M)	(0.60, 0.30, 0.10)
Significant (S)	(0.80, 0.15, 0.05)
High (H)	(0.95, 0.05, 0.00)

Table A2
Assessments for Indicators.

Expert 1	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF		S	M	M
WASTMNG	H		H	S
PROLFCYC	M	H		H
ECOFEA	S	M	H	
Expert 2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF		M	M	M
WASTMNG	H		L	M
PROLFCYC	H	H		M
ECOFEA	S	S	M	
Expert 3	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF		H	H	H
WASTMNG	H		M	M
PROLFCYC	S	S		M
ECOFEA	M	M	S	

Table A3
Assessments for Strategies.

Expert 1	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	H	S	M	M
ECODSG	S	S	H	S
EXCECS	H	H	M	H
INFCSDM	H	M	S	H
INVTEC	S	H	M	M
Expert 2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	H	H	M	H
ECODSG	M	M	H	H
EXCECS	H	H	M	H
INFCSDM	H	M	S	H
INVTEC	H	H	H	H
Expert 3	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	H	S	H	H
ECODSG	S	S	H	S
EXCECS	H	H	H	H
INFCSDM	H	H	S	H
INVTEC	H	H	S	H

Table A4
Demographic Information of Experts.

	Age	Experience in total (year)	Education	Profession	Papers	Experience	Certifications	Teaching	Conference	Patents
Expert 1	44	18	PhD	Professor	30	2	4	18	46	1
Expert 2	42	14	PhD	Board Member	8	6	7	3	8	1
Expert 3	50	25	PhD	Chief of Executive Officer	5	8	5	1	25	3

Table A5

Normalized Values of Demographic Information and Weights of Experts.

	Age	Experience	Education	Profession	Papers	Experience	Certifications	Teaching	Conference	Patents	Average Scores	Weights
Expert 1	0.88	0.72	1.00	0.90	1.00	0.25	0.57	1.00	1.00	0.33	0.77	0.364
Expert 2	0.84	0.56	1.00	0.80	0.27	0.75	1.00	0.17	0.17	0.33	0.59	0.280
Expert 3	1.00	1.00	1.00	1.00	0.17	1.00	0.71	0.06	0.54	1.00	0.75	0.356

Table A6

Reward Degrees for Relation Matrix.

Expert 1-2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.00, 0.00, 0.00)	(-0.06, 0.04, 0.01)	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)
WASTMNG	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(-0.15, 0.13, 0.03)	(-0.06, 0.04, 0.01)
PROLFCYC	(0.10, -0.07, -0.03)	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(-0.10, 0.07, 0.03)
ECOFEA	(0.00, 0.00, 0.00)	(0.06, -0.04, -0.01)	(-0.10, 0.07, 0.03)	(0.00, 0.00, 0.00)
Expert 1-3	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.00, 0.00, 0.00)	(0.05, -0.04, -0.02)	(0.12, -0.09, -0.04)	(0.12, -0.09, -0.04)
WASTMNG	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(-0.12, 0.09, 0.04)	(-0.07, 0.05, 0.02)
PROLFCYC	(0.07, -0.05, -0.02)	(-0.05, 0.04, 0.02)	(0.00, 0.00, 0.00)	(-0.12, 0.09, 0.04)
ECOFEA	(-0.07, 0.05, 0.02)	(0.00, 0.00, 0.00)	(-0.05, 0.04, 0.02)	(0.00, 0.00, 0.00)

Table A7

Reward Degrees for Decision Matrix.

Expert 1-Expert 2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	(0.00, 0.00, 0.00)	(0.04, -0.03, -0.01)	(0.00, 0.00, 0.00)	(0.10, -0.07, -0.03)
ECODSG	(-0.06, 0.04, 0.01)	(-0.06, 0.04, 0.01)	(0.00, 0.00, 0.00)	(0.04, -0.03, -0.01)
EXCECS	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)
INFCSDM	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)
INVTEC	(0.04, -0.03, -0.01)	(0.00, 0.00, 0.00)	(0.10, -0.07, -0.03)	(0.10, -0.07, -0.03)
Expert 1-Expert 3	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(0.12, -0.09, -0.04)	(0.12, -0.09, -0.04)
ECODSG	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)
EXCECS	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(0.12, -0.09, -0.04)	(0.00, 0.00, 0.00)
INFCSDM	(0.00, 0.00, 0.00)	(0.12, -0.09, -0.04)	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)
INVTEC	(0.05, -0.04, -0.02)	(0.00, 0.00, 0.00)	(0.07, -0.05, -0.02)	(0.12, -0.09, -0.04)

Table A8

Penalty Degrees for Relation Matrix.

Expert 1-2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.00, 0.00, 0.00)	(0.07, -0.05, -0.02)	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)
WASTMNG	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(0.20, -0.16, -0.04)	(0.07, -0.05, -0.02)
PROLFCYC	(-0.13, 0.09, 0.04)	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(0.13, -0.09, -0.04)
ECOFEA	(0.00, 0.00, 0.00)	(-0.07, 0.05, 0.02)	(0.13, -0.09, -0.04)	(0.00, 0.00, 0.00)
Expert 1-3	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.00, 0.00, 0.00)	(-0.05, 0.04, 0.02)	(-0.13, 0.09, 0.04)	(-0.13, 0.09, 0.04)
WASTMNG	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(0.13, -0.09, -0.04)	(0.07, -0.05, -0.02)
PROLFCYC	(-0.07, 0.05, 0.02)	(0.05, -0.04, -0.02)	(0.00, 0.00, 0.00)	(0.13, -0.09, -0.04)
ECOFEA	(0.07, -0.05, -0.02)	(0.00, 0.00, 0.00)	(0.05, -0.04, -0.02)	(0.00, 0.00, 0.00)

Table A9

Penalty Degrees for Decision Matrix.

Expert 1-2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	(0.00, 0.00, 0.00)	(-0.05, 0.04, 0.02)	(0.00, 0.00, 0.00)	(-0.13, 0.09, 0.04)
ECODSG	(0.07, -0.05, -0.02)	(0.07, -0.05, -0.02)	(0.00, 0.00, 0.00)	(-0.05, 0.04, 0.02)
EXCECS	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)
INFCSDM	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)
INVTEC	(-0.05, 0.04, 0.02)	(0.00, 0.00, 0.00)	(-0.13, 0.09, 0.04)	(-0.13, 0.09, 0.04)
Expert 1-3	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(-0.13, 0.09, 0.04)	(-0.13, 0.09, 0.04)

(continued on next page)

Table A9 (continued)

Expert 1–2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ECODSG	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)
EXCECS	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)	(-0.13, 0.09, 0.04)	(0.00, 0.00, 0.00)
INFCSDM	(0.00, 0.00, 0.00)	(-0.13, 0.09, 0.04)	(0.00, 0.00, 0.00)	(0.00, 0.00, 0.00)
INVTEC	(-0.05, 0.04, 0.02)	(0.00, 0.00, 0.00)	(-0.07, 0.05, 0.02)	(-0.13, 0.09, 0.04)

Table A10

Relation Matrix.

Expert 1-Expert 2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.00, 0.00, 0.00)	(0.79, 0.16, 0.05)	(0.60, 0.30, 0.10)	(0.60, 0.30, 0.10)
WASTMNG	(0.95, 0.05, 0.00)	(0.00, 0.00, 0.00)	(0.91, 0.08, 0.01)	(0.79, 0.16, 0.05)
PROLFCYC	(0.62, 0.28, 0.09)	(0.95, 0.05, 0.00)	(0.00, 0.00, 0.00)	(0.93, 0.07, 0.01)
ECOFEA	(0.80, 0.15, 0.05)	(0.61, 0.29, 0.10)	(0.93, 0.07, 0.01)	(0.00, 0.00, 0.00)
Expert 1-Expert 3	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.00, 0.00, 0.00)	(0.81, 0.14, 0.05)	(0.63, 0.28, 0.09)	(0.63, 0.28, 0.09)
WASTMNG	(0.95, 0.05, 0.00)	(0.00, 0.00, 0.00)	(0.92, 0.07, 0.01)	(0.79, 0.16, 0.05)
PROLFCYC	(0.61, 0.29, 0.10)	(0.94, 0.06, 0.00)	(0.00, 0.00, 0.00)	(0.92, 0.07, 0.01)
ECOFEA	(0.79, 0.16, 0.05)	(0.60, 0.30, 0.10)	(0.94, 0.06, 0.00)	(0.00, 0.00, 0.00)

Table A11

Decision Matrix.

Expert 1-Expert 2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	(0.95, 0.05, 0.00)	(0.81, 0.14, 0.05)	(0.60, 0.30, 0.10)	(0.62, 0.28, 0.09)
ECODSG	(0.79, 0.16, 0.05)	(0.79, 0.16, 0.05)	(0.95, 0.05, 0.00)	(0.81, 0.14, 0.05)
EXCECS	(0.95, 0.05, 0.00)	(0.95, 0.05, 0.00)	(0.60, 0.30, 0.10)	(0.95, 0.05, 0.00)
INFCSDM	(0.95, 0.05, 0.00)	(0.60, 0.30, 0.10)	(0.80, 0.15, 0.05)	(0.95, 0.05, 0.00)
INVTEC	(0.81, 0.14, 0.05)	(0.95, 0.05, 0.00)	(0.62, 0.28, 0.09)	(0.62, 0.28, 0.09)
Expert 1-Expert 3	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	(0.95, 0.05, 0.00)	(0.80, 0.15, 0.05)	(0.63, 0.28, 0.09)	(0.63, 0.28, 0.09)
ECODSG	(0.80, 0.15, 0.05)	(0.80, 0.15, 0.05)	(0.95, 0.05, 0.00)	(0.80, 0.15, 0.05)
EXCECS	(0.95, 0.05, 0.00)	(0.95, 0.05, 0.00)	(0.63, 0.28, 0.09)	(0.95, 0.05, 0.00)
INFCSDM	(0.95, 0.05, 0.00)	(0.63, 0.28, 0.09)	(0.80, 0.15, 0.05)	(0.95, 0.05, 0.00)
INVTEC	(0.81, 0.14, 0.05)	(0.95, 0.05, 0.00)	(0.61, 0.29, 0.10)	(0.63, 0.28, 0.09)

Table A12

Difference of Relation Matrix.

Expert 1-Expert 2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.000,000,000)	(0.013,010,003)	(0.000,000,000)	(0.000,000,000)
WASTMNG	(0.000,000,000)	(0.000,000,000)	(0.035,029,006)	(0.013,010,003)
PROLFCYC	(0.023,016,006)	(0.000,000,000)	(0.000,000,000)	(0.023,016,006)
ECOFEA	(0.000,000,000)	(0.013,010,003)	(0.023,016,006)	(0.000,000,000)
Expert 1-Expert 3	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.000,000,000)	(0.011,007,004)	(0.025,018,007)	(0.025,018,007)
WASTMNG	(0.000,000,000)	(0.000,000,000)	(0.025,018,007)	(0.014,011,004)
PROLFCYC	(0.014,011,004)	(0.011,007,004)	(0.000,000,000)	(0.025,018,007)
ECOFEA	(0.014,011,004)	(0.000,000,000)	(0.011,007,004)	(0.000,000,000)

Table A13

Difference of Decision Matrix.

Expert 1-Expert 2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	(0.000,000,000)	(0.010,006,003)	(0.000,000,000)	(0.023,016,006)
ECODSG	(0.013,010,003)	(0.013,010,003)	(0.000,000,000)	(0.010,006,003)
EXCECS	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)
INFCSDM	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)
INVTEC	(0.010,006,003)	(0.000,000,000)	(0.023,016,006)	(0.023,016,006)
Expert 1-Expert 3	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	(0.000,000,000)	(0.000,000,000)	(0.025,018,007)	(0.025,018,007)
ECODSG	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)
EXCECS	(0.000,000,000)	(0.000,000,000)	(0.025,018,007)	(0.000,000,000)
INFCSDM	(0.000,000,000)	(0.025,018,007)	(0.000,000,000)	(0.000,000,000)
INVTEC	(0.011,007,004)	(0.000,000,000)	(0.014,011,004)	(0.025,018,007)

Table A14
Differences of Relation Matrix Among Expert 1–2.

Iteration 2 Expert 1–2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.000,000,000)	(0.012,009,003)	(0.000,000,000)	(0.000,000,000)
WASTMNG	(0.000,000,000)	(0.000,000,000)	(0.033,027,006)	(0.012,009,003)
PROLFCYC	(0.021,015,006)	(0.000,000,000)	(0.000,000,000)	(0.021,015,006)
ECOFEA	(0.000,000,000)	(0.012,009,003)	(0.021,015,006)	(0.000,000,000)
Iteration 3 Expert 1–2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.000,000,000)	(0.011,008,003)	(0.000,000,000)	(0.000,000,000)
WASTMNG	(0.000,000,000)	(0.000,000,000)	(0.031,025,006)	(0.011,008,003)
PROLFCYC	(0.020,014,006)	(0.000,000,000)	(0.000,000,000)	(0.020,014,006)
ECOFEA	(0.000,000,000)	(0.011,008,003)	(0.020,014,006)	(0.000,000,000)
Iteration 4 Expert 1–2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.000,000,000)	(0.011,008,003)	(0.000,000,000)	(0.000,000,000)
WASTMNG	(0.000,000,000)	(0.000,000,000)	(0.029,024,005)	(0.011,008,003)
PROLFCYC	(0.018,013,005)	(0.000,000,000)	(0.000,000,000)	(0.018,013,005)
ECOFEA	(0.000,000,000)	(0.011,008,003)	(0.018,013,005)	(0.000,000,000)
Iteration 5 Expert 1–2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.000,000,000)	(0.010,007,002)	(0.000,000,000)	(0.000,000,000)
WASTMNG	(0.000,000,000)	(0.000,000,000)	(0.027,022,005)	(0.010,007,002)
PROLFCYC	(0.017,012,005)	(0.000,000,000)	(0.000,000,000)	(0.017,012,005)
ECOFEA	(0.000,000,000)	(0.010,007,002)	(0.017,012,005)	(0.000,000,000)
Iteration 6 Expert 1–2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.000,000,000)	(0.009,007,002)	(0.000,000,000)	(0.000,000,000)
WASTMNG	(0.000,000,000)	(0.000,000,000)	(0.025,021,005)	(0.009,007,002)
PROLFCYC	(0.016,012,005)	(0.000,000,000)	(0.000,000,000)	(0.016,012,005)
ECOFEA	(0.000,000,000)	(0.009,007,002)	(0.016,012,005)	(0.000,000,000)
Iteration 7 Expert 1–2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.000,000,000)	(0.009,006,002)	(0.000,000,000)	(0.000,000,000)
WASTMNG	(0.000,000,000)	(0.000,000,000)	(0.024,019,004)	(0.009,006,002)
PROLFCYC	(0.015,011,004)	(0.000,000,000)	(0.000,000,000)	(0.015,011,004)
ECOFEA	(0.000,000,000)	(0.009,006,002)	(0.015,011,004)	(0.000,000,000)
Iteration 8 Expert 1–2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.000,000,000)	(0.008,006,002)	(0.000,000,000)	(0.000,000,000)
WASTMNG	(0.000,000,000)	(0.000,000,000)	(0.020,018,004)	(0.008,006,002)
PROLFCYC	(0.014,010,004)	(0.000,000,000)	(0.000,000,000)	(0.014,010,004)
ECOFEA	(0.000,000,000)	(0.008,006,002)	(0.014,010,004)	(0.000,000,000)

Table A15
Differences of Relation Matrix Among Expert 1–3.

Iteration 2 Expert 1–3	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.000,000,000)	(0.010,007,003)	(0.023,017,007)	(0.023,017,007)
WASTMNG	(0.000,000,000)	(0.000,000,000)	(0.023,017,007)	(0.013,010,003)
PROLFCYC	(0.013,010,003)	(0.010,007,003)	(0.000,000,000)	(0.023,017,007)
ECOFEA	(0.013,010,003)	(0.000,000,000)	(0.010,007,003)	(0.000,000,000)
Iteration 3 Expert 1–3	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.000,000,000)	(0.009,006,003)	(0.022,015,006)	(0.022,015,006)
WASTMNG	(0.000,000,000)	(0.000,000,000)	(0.022,015,006)	(0.012,009,003)
PROLFCYC	(0.012,009,003)	(0.009,006,003)	(0.000,000,000)	(0.022,015,006)
ECOFEA	(0.012,009,003)	(0.000,000,000)	(0.009,006,003)	(0.000,000,000)
Iteration 4 Expert 1–3	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.000,000,000)	(0.009,006,003)	(0.020,014,006)	(0.020,014,006)
WASTMNG	(0.000,000,000)	(0.000,000,000)	(0.020,014,006)	(0.012,009,003)
PROLFCYC	(0.012,009,003)	(0.009,006,003)	(0.000,000,000)	(0.020,014,006)
ECOFEA	(0.012,009,003)	(0.000,000,000)	(0.009,006,003)	(0.000,000,000)
Iteration 5 Expert 1–3	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.000,000,000)	(0.008,005,003)	(0.019,013,005)	(0.019,013,005)
WASTMNG	(0.000,000,000)	(0.000,000,000)	(0.019,013,005)	(0.011,008,003)
PROLFCYC	(0.011,008,003)	(0.008,005,003)	(0.000,000,000)	(0.019,013,005)
ECOFEA	(0.011,008,003)	(0.000,000,000)	(0.008,005,003)	(0.000,000,000)

Table A16
Differences of Decision Matrix Among Expert 1–2.

Iteration 2 Expert 1–2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	(0.000,000,000)	(0.009,006,003)	(0.000,000,000)	(0.021,015,006)
ECODSG	(0.012,009,003)	(0.012,009,003)	(0.000,000,000)	(0.009,006,003)
EXCECS	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)
INFCSDM	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)
INVTEC	(0.009,006,003)	(0.000,000,000)	(0.021,015,006)	(0.021,015,006)
Iteration 3 Expert 1–2	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	(0.000,000,000)	(0.008,006,003)	(0.000,000,000)	(0.020,014,006)
ECODSG	(0.011,008,003)	(0.011,008,003)	(0.000,000,000)	(0.008,006,003)
EXCECS	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)
INFCSDM	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)
INVTEC	(0.008,006,003)	(0.000,000,000)	(0.020,014,006)	(0.020,014,006)

Table A17
Differences of Decision Matrix Among Expert 1–3.

Iteration 2 Expert 1–3	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	(0.000,000,000)	(0.000,000,000)	(0.023,017,007)	(0.023,017,007)
ECODSG	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)
EXCECS	(0.000,000,000)	(0.000,000,000)	(0.023,017,007)	(0.000,000,000)
INFCSDM	(0.000,000,000)	(0.023,017,007)	(0.000,000,000)	(0.000,000,000)
INVTEC	(0.010,007,003)	(0.000,000,000)	(0.013,010,003)	(0.023,017,007)
Iteration 3 Expert 1–3	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	(0.000,000,000)	(0.000,000,000)	(0.022,015,006)	(0.022,015,006)
ECODSG	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)
EXCECS	(0.000,000,000)	(0.000,000,000)	(0.022,015,006)	(0.000,000,000)
INFCSDM	(0.000,000,000)	(0.022,015,006)	(0.000,000,000)	(0.000,000,000)
INVTEC	(0.009,006,003)	(0.000,000,000)	(0.012,009,003)	(0.022,015,006)
Iteration 4 Expert 1–3	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	(0.000,000,000)	(0.000,000,000)	(0.019,013,005)	(0.019,013,005)
ECODSG	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)	(0.000,000,000)
EXCECS	(0.000,000,000)	(0.000,000,000)	(0.019,013,005)	(0.000,000,000)
INFCSDM	(0.000,000,000)	(0.019,013,005)	(0.000,000,000)	(0.000,000,000)
INVTEC	(0.008,005,003)	(0.000,000,000)	(0.011,008,003)	(0.019,013,005)

Table A18
Q Values of Relation Matrix.

Expert 1 (initial Q matrix)	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.00,00,00)	(0.80,15,05)	(0.60,30,10)	(0.60,30,10)
WASTMNG	(0.95,05,00)	(0.00,00,00)	(0.95,05,00)	(0.80,15,05)
PROLFCYC	(0.60,30,10)	(0.95,05,00)	(0.00,00,00)	(0.95,05,00)
ECOFEA	(0.80,15,05)	(0.60,30,10)	(0.95,05,00)	(0.00,00,00)
Expert 2 (Balanced Q matrix)	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.00,00,00)	(0.70,22,07)	(0.60,30,10)	(0.60,30,10)
WASTMNG	(0.95,05,00)	(0.00,00,00)	(0.68,27,05)	(0.70,22,07)
PROLFCYC	(0.77,18,05)	(0.95,05,00)	(0.00,00,00)	(0.78,17,05)
ECOFEA	(0.80,15,05)	(0.70,23,08)	(0.78,17,05)	(0.00,00,00)
Expert 3 (Balanced Q matrix)	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.00,00,00)	(0.85,12,03)	(0.71,22,07)	(0.71,22,07)
WASTMNG	(0.95,05,00)	(0.00,00,00)	(0.84,13,03)	(0.74,20,07)
PROLFCYC	(0.66,25,08)	(0.90,08,02)	(0.00,00,00)	(0.84,13,03)
ECOFEA	(0.74,20,07)	(0.60,30,10)	(0.90,08,02)	(0.00,00,00)

Table A19
Q Values for Decision Matrix.

Expert 1 (initial Q matrix)	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	(0.95,05,00)	(0.80,15,05)	(0.60,30,10)	(0.60,30,10)
ECODSG	(0.80,15,05)	(0.80,15,05)	(0.95,05,00)	(0.80,15,05)
EXCECS	(0.95,05,00)	(0.95,05,00)	(0.60,30,10)	(0.95,05,00)
INFCSDM	(0.95,05,00)	(0.60,30,10)	(0.80,15,05)	(0.95,05,00)

(continued on next page)

Table A19 (continued)

Expert 1 (initial Q matrix)	ENEFF	WASTMNG	PROLFCYC	ECOFEA
INVTEC	(0.80,.15,.05)	(0.95,.05,.00)	(0.60,.30,.10)	(0.60,.30,.10)
Expert 2 (Balanced Q matrix)	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	(0.95,.05,.00)	(0.83,.13,.04)	(0.60,.30,.10)	(0.66,.25,.08)
ECODSG	(0.76,.18,.06)	(0.76,.18,.06)	(0.95,.05,.00)	(0.83,.13,.04)
EXCECS	(0.95,.05,.00)	(0.95,.05,.00)	(0.60,.30,.10)	(0.95,.05,.00)
INFCSDM	(0.95,.05,.00)	(0.60,.30,.10)	(0.80,.15,.05)	(0.95,.05,.00)
INVTEC	(0.83,.13,.04)	(0.95,.05,.00)	(0.66,.25,.08)	(0.66,.25,.08)
Expert 3 (Balanced Q matrix)	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	(0.95,.05,.00)	(0.80,.15,.05)	(0.71,.22,.07)	(0.71,.22,.07)
ECODSG	(0.80,.15,.05)	(0.80,.15,.05)	(0.95,.05,.00)	(0.80,.15,.05)
EXCECS	(0.95,.05,.00)	(0.95,.05,.00)	(0.71,.22,.07)	(0.95,.05,.00)
INFCSDM	(0.95,.05,.00)	(0.71,.22,.07)	(0.80,.15,.05)	(0.95,.05,.00)
INVTEC	(0.85,.12,.03)	(0.95,.05,.00)	(0.66,.25,.08)	(0.71,.22,.07)

Table A20

Averaged Fuzzy Numbers.

	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	(0.00, 0.00, 0.00)	(0.78, 0.16, 0.05)	(0.64, 0.27, 0.09)	(0.64, 0.27, 0.09)
WASTMNG	(0.95, 0.05, 0.00)	(0.00, 0.00, 0.00)	(0.82, 0.15, 0.03)	(0.75, 0.19, 0.06)
PROLFCYC	(0.68, 0.24, 0.08)	(0.93, 0.06, 0.01)	(0.00, 0.00, 0.00)	(0.86, 0.12, 0.03)
ECOFEA	(0.78, 0.17, 0.06)	(0.63, 0.28, 0.09)	(0.88, 0.10, 0.02)	(0.00, 0.00, 0.00)

Table A21

Fuzzy Vectors.

Vectors	Fuzzy Vectors
u_1	[(0.78, 0.16, 0.05), (0.64, 0.27, 0.09), (0.64, 0.27, 0.09)]
u_2	[(0.95, 0.05, 0.00), (0.82, 0.15, 0.03), (0.75, 0.19, 0.06)]
u_3	[(0.68, 0.24, 0.08), (0.93, 0.06, 0.01), (0.86, 0.12, 0.03)]
u_4	[(0.78, 0.17, 0.06), (0.63, 0.28, 0.09), (0.88, 0.10, 0.02)]

Table A22

Dot Products.

	u_1	u_2	u_3	u_4
u_1		1.853	1.766	1.715
u_2	1.853		2.099	1.989
u_3	1.766	2.099		1.945
u_4	1.715	1.989	1.945	

Table A23

Cosine Values.

	$\cos(\theta_{u_1})$	$\cos(\theta_{u_2})$	$\cos(\theta_{u_3})$	$\cos(\theta_{u_4})$
$\cos(\theta_{u_1})$		0.981	0.946	0.977
$\cos(\theta_{u_2})$	0.981		0.964	0.971
$\cos(\theta_{u_3})$	0.946	0.964		0.962
$\cos(\theta_{u_4})$	0.977	0.971	0.962	

Table A24

Angle In Radians.

	θ_{u_1}	θ_{u_2}	θ_{u_3}	θ_{u_4}
θ_{u_1}		0.197	0.329	0.216
θ_{u_2}	0.197		0.269	0.241
θ_{u_3}	0.329	0.269		0.278
θ_{u_4}	0.216	0.241	0.278	

Table A25
Normalized Values.

Linear	$norm(\theta_{u_1})$	$norm(\theta_{u_2})$	$norm(\theta_{u_3})$	$norm(\theta_{u_4})$
$norm(\theta_{u_1})$	0.000	0.063	0.105	0.069
$norm(\theta_{u_2})$	0.063	0.000	0.086	0.077
$norm(\theta_{u_3})$	0.105	0.086	0.000	0.088
$norm(\theta_{u_4})$	0.069	0.077	0.088	0.000
Trigonal Planar	$norm(\theta_{u_1})$	$norm(\theta_{u_2})$	$norm(\theta_{u_3})$	$norm(\theta_{u_4})$
$norm(\theta_{u_1})$	0.000	0.094	0.157	0.103
$norm(\theta_{u_2})$	0.094	0.000	0.129	0.115
$norm(\theta_{u_3})$	0.157	0.129	0.000	0.133
$norm(\theta_{u_4})$	0.103	0.115	0.133	0.000
Tetrahedral	$norm(\theta_{u_1})$	$norm(\theta_{u_2})$	$norm(\theta_{u_3})$	$norm(\theta_{u_4})$
$norm(\theta_{u_1})$	0.000	0.103	0.172	0.113
$norm(\theta_{u_2})$	0.103	0.000	0.141	0.126
$norm(\theta_{u_3})$	0.172	0.141	0.000	0.146
$norm(\theta_{u_4})$	0.113	0.126	0.146	0.000
Trigonal Bipyramidal	$norm(\theta_{u_1})$	$norm(\theta_{u_2})$	$norm(\theta_{u_3})$	$norm(\theta_{u_4})$
$norm(\theta_{u_1})$	0.000	0.108	0.180	0.118
$norm(\theta_{u_2})$	0.108	0.000	0.147	0.132
$norm(\theta_{u_3})$	0.180	0.147	0.000	0.152
$norm(\theta_{u_4})$	0.118	0.132	0.152	0.000
Octahedral	$norm(\theta_{u_1})$	$norm(\theta_{u_2})$	$norm(\theta_{u_3})$	$norm(\theta_{u_4})$
$norm(\theta_{u_1})$	0.000	0.126	0.209	0.137
$norm(\theta_{u_2})$	0.126	0.000	0.171	0.153
$norm(\theta_{u_3})$	0.209	0.171	0.000	0.177
$norm(\theta_{u_4})$	0.137	0.153	0.177	0.000

Table A26
Reciprocal Values.

	$\frac{1}{norm(\theta_{u_1})}$	$\frac{1}{norm(\theta_{u_2})}$	$\frac{1}{norm(\theta_{u_3})}$	$\frac{1}{norm(\theta_{u_4})}$
$\frac{1}{norm(\theta_{u_1})}$		15.915	9.560	14.556
$\frac{1}{norm(\theta_{u_2})}$	15.915		11.668	13.033
$\frac{1}{norm(\theta_{u_3})}$	9.560	11.668		11.303
$\frac{1}{norm(\theta_{u_4})}$	14.556	13.033	11.303	

Table A27
Normalized Matrix.

	ENEFF	WASTMNG	PROLFCYC	ECOFEA
ENEFF	0	0.209	0.126	0.191
WASTMNG	0.209	0	0.153	0.171
PROLFCYC	0.126	0.153	0	0.149
ECOFEA	0.191	0.171	0.149	0

Table A28
Averaged Fuzzy Numbers.

	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	(0.95, 0.05, 0.00)	(0.81, 0.14, 0.05)	(0.64, 0.27, 0.09)	(0.66, 0.26, 0.08)
ECODSG	(0.79, 0.16, 0.05)	(0.79, 0.16, 0.05)	(0.95, 0.05, 0.00)	(0.81, 0.14, 0.05)
EXCECS	(0.95, 0.05, 0.00)	(0.95, 0.05, 0.00)	(0.64, 0.27, 0.09)	(0.95, 0.05, 0.00)
INFCSDM	(0.95, 0.05, 0.00)	(0.64, 0.27, 0.09)	(0.80, 0.15, 0.05)	(0.95, 0.05, 0.00)
INVTEC	(0.82, 0.13, 0.04)	(0.95, 0.05, 0.00)	(0.64, 0.27, 0.09)	(0.66, 0.26, 0.08)

Table A29
Weighted Decision Matrix.

	ENEFF	WASTMNG	PROLFCYC	ECOFEA
OFFSRCS	(0.24, 0.01, 0.00)	(0.20, 0.04, 0.01)	(0.16, 0.07, 0.02)	(0.16, 0.06, 0.02)
ECODSG	(0.20, 0.04, 0.01)	(0.20, 0.04, 0.01)	(0.23, 0.01, 0.00)	(0.20, 0.04, 0.01)
EXCECS	(0.24, 0.01, 0.00)	(0.24, 0.01, 0.00)	(0.16, 0.07, 0.02)	(0.24, 0.01, 0.00)
INFCSDM	(0.24, 0.01, 0.00)	(0.16, 0.07, 0.02)	(0.20, 0.04, 0.01)	(0.24, 0.01, 0.00)
INVTEC	(0.21, 0.03, 0.01)	(0.24, 0.01, 0.00)	(0.16, 0.07, 0.02)	(0.16, 0.06, 0.02)

Table A30
Fuzzy Vectors.

Vectors	Fuzzy Vectors
y_1	[(0.24, 0.01, 0.00), (0.20, 0.04, 0.01), (0.16, 0.07, 0.02), (0.16, 0.06, 0.02)]
y_2	[(0.20, 0.04, 0.01), (0.20, 0.04, 0.01), (0.23, 0.01, 0.00), (0.20, 0.04, 0.01)]
y_3	[(0.24, 0.01, 0.00), (0.24, 0.01, 0.00), (0.16, 0.07, 0.02), (0.24, 0.01, 0.00)]
y_4	[(0.24, 0.01, 0.00), (0.16, 0.07, 0.02), (0.20, 0.04, 0.01), (0.24, 0.01, 0.00)]
y_5	[(0.21, 0.03, 0.01), (0.24, 0.01, 0.00), (0.16, 0.07, 0.02), (0.16, 0.06, 0.02)]

Table A31
Dot Products.

	y_1	y_2	y_3	y_4	y_5
y_1		0.163	0.176	0.166	0.161
y_2	0.163		0.182	0.178	0.164
y_3	0.176	0.182		0.187	0.177
y_4	0.166	0.178	0.187		0.163
y_5	0.161	0.164	0.177	0.163	

Table A32
Cosine of Angle.

	$\cos(\theta_{y_1})$	$\cos(\theta_{y_2})$	$\cos(\theta_{y_3})$	$\cos(\theta_{y_4})$	$\cos(\theta_{y_5})$
$\cos(\theta_{y_1})$		0.959	0.977	0.961	0.989
$\cos(\theta_{y_2})$	0.959		0.958	0.975	0.960
$\cos(\theta_{y_3})$	0.977	0.958		0.968	0.978
$\cos(\theta_{y_4})$	0.961	0.975	0.968		0.937
$\cos(\theta_{y_5})$	0.989	0.960	0.978	0.937	

Table A33
Angle in Radians for Decision Matrix.

	θ_{y_1}	θ_{y_2}	θ_{y_3}	θ_{y_4}	θ_{y_5}
θ_{y_1}		0.287	0.216	0.280	0.147
θ_{y_2}	0.287		0.291	0.222	0.285
θ_{y_3}	0.216	0.291		0.253	0.212
θ_{y_4}	0.280	0.222	0.253		0.356
θ_{y_5}	0.147	0.285	0.212	0.356	

Table A34
Normalized Values.

Linear	$norm(\theta_{y_1})$	$norm(\theta_{y_2})$	$norm(\theta_{y_3})$	$norm(\theta_{y_4})$	$norm(\theta_{y_5})$
$norm(\theta_{y_1})$	0.000	0.092	0.069	0.089	0.047
$norm(\theta_{y_2})$	0.092	0.000	0.093	0.071	0.091
$norm(\theta_{y_3})$	0.069	0.093	0.000	0.080	0.068
$norm(\theta_{y_4})$	0.089	0.071	0.080	0.000	0.113
$norm(\theta_{y_5})$	0.047	0.091	0.068	0.113	0.000
Trigonal Planar	$norm(\theta_{y_1})$	$norm(\theta_{y_2})$	$norm(\theta_{y_3})$	$norm(\theta_{y_4})$	$norm(\theta_{y_5})$
$norm(\theta_{y_1})$	0.000	0.137	0.103	0.134	0.070
$norm(\theta_{y_2})$	0.137	0.000	0.139	0.106	0.136

(continued on next page)

Table A34 (continued)

Linear	$norm(\theta_{y_1})$	$norm(\theta_{y_2})$	$norm(\theta_{y_3})$	$norm(\theta_{y_4})$	$norm(\theta_{y_5})$
$norm(\theta_{y_3})$	0.103	0.139	0.000	0.121	0.101
$norm(\theta_{y_4})$	0.134	0.106	0.121	0.000	0.170
$norm(\theta_{y_5})$	0.070	0.136	0.101	0.170	0.000
Tetrahedral	$norm(\theta_{y_1})$	$norm(\theta_{y_2})$	$norm(\theta_{y_3})$	$norm(\theta_{y_4})$	$norm(\theta_{y_5})$
$norm(\theta_{y_1})$	0.000	0.151	0.113	0.147	0.077
$norm(\theta_{y_2})$	0.151	0.000	0.152	0.116	0.149
$norm(\theta_{y_3})$	0.113	0.152	0.000	0.132	0.111
$norm(\theta_{y_4})$	0.147	0.116	0.132	0.000	0.186
$norm(\theta_{y_5})$	0.077	0.149	0.111	0.186	0.000
Trigonal Bipyramidal	$norm(\theta_{y_1})$	$norm(\theta_{y_2})$	$norm(\theta_{y_3})$	$norm(\theta_{y_4})$	$norm(\theta_{y_5})$
$norm(\theta_{y_1})$	0.000	0.157	0.118	0.153	0.080
$norm(\theta_{y_2})$	0.157	0.000	0.159	0.122	0.156
$norm(\theta_{y_3})$	0.118	0.159	0.000	0.138	0.116
$norm(\theta_{y_4})$	0.153	0.122	0.138	0.000	0.195
$norm(\theta_{y_5})$	0.080	0.156	0.116	0.195	0.000
Octahedral	$norm(\theta_{y_1})$	$norm(\theta_{y_2})$	$norm(\theta_{y_3})$	$norm(\theta_{y_4})$	$norm(\theta_{y_5})$
$norm(\theta_{y_1})$	0.000	0.183	0.137	0.178	0.094
$norm(\theta_{y_2})$	0.183	0.000	0.185	0.142	0.181
$norm(\theta_{y_3})$	0.137	0.185	0.000	0.161	0.135
$norm(\theta_{y_4})$	0.178	0.142	0.161	0.000	0.227
$norm(\theta_{y_5})$	0.094	0.181	0.135	0.227	0.000

Table A35
Reciprocal Values.

	$\frac{1}{norm(\theta_{y_1})}$	$\frac{1}{norm(\theta_{y_2})}$	$\frac{1}{norm(\theta_{y_3})}$	$\frac{1}{norm(\theta_{y_4})}$	$\frac{1}{norm(\theta_{y_5})}$
$\frac{1}{norm(\theta_{y_1})}$		10.928	14.557	11.223	21.357
$\frac{1}{norm(\theta_{y_2})}$	10.928		10.796	14.124	11.020
$\frac{1}{norm(\theta_{y_3})}$	14.557	10.796		12.432	14.791
$\frac{1}{norm(\theta_{y_4})}$	11.223	14.124	12.432		8.825
$\frac{1}{norm(\theta_{y_5})}$	21.357	11.020	14.791	8.825	

Table A36
Symbols.

Symbol	Explanation
μ	Membership
ν	Non-membership
ξ	Hesitancy
θ	Angle
λ	Scaler
Q	Evaluation matrix
R	Reward degree
r	Reward factor
P	Penalty degree
p	Penalty factor
α	Learning rate
$Q_{s,a}$	Updated evaluations
Δ	Absolute differences
Δ_{max}	Maximum of absolute differences
ζ	Aggregated fuzzy value
u	Fuzzy relation vector
NR	Normalization of relation matrix
W	Weight of criteria
B	Weighted decision matrix
y	Fuzzy decision vector
\cdot	Dot product
f	Final matrix
Agg	Aggregated value

References

- [1] Ling TS, Abidin MNZ, Ma'amor, A., Hashim, N. A., & Hashim, M. L. H.. Polyethylene terephthalate membrane: a review of fabrication techniques, separation processes, and modifications. *Separation and Purification Technology*; 2024.
- [2] Soni A, Chakraborty S, Das PK, Saha AK. Selection of sustainable construction material from recycled waste plastics by q-rung orthopair fuzzy SWARA-MABAC approach. *Chemosphere* 2024;143166.
- [3] Najjar IN, Sharma P, Das R, Mondal K, Singh AK, Tamang S, et al. In search of Poly-3-Hydroxybutyrate (PHB): a Comprehensive Review Unveiling applications and Progress in Fostering a Sustainable Bio-Circular Economy. *Food Bioprod. Process.* 2024.
- [4] Valt RBG, Pires CMG, Ponte MJDS, Ponte HDA. Lanthanum removal from spent catalysts of refinery applying electroremediation technique coupled to selective precipitation. *Braz. J. Chem. Eng.* 2024;1–13.
- [5] Castillo García P, Fernández-Rodríguez MJ, Borja R, Mancilla-Leytón JM, de la Lama-Calvente D. Research Trends in the Recovery of By-Products from Organic Waste Treated by Anaerobic Digestion: a 30-Year Bibliometric Analysis. *Fermentation* 2024;10(9):446.
- [6] Gurung SK, Mickan BS, Middleton JA, Singh PK, Jenkins SN, Rengel Z, et al. Manure-derived black soldier fly frass enhanced the growth of chilli plants (*Capsicum annuum* L.) and altered rhizosphere bacterial community. *Appl. Soil Ecol.* 2024;202:105605.
- [7] Moslem S. A novel parsimonious spherical fuzzy analytic hierarchy process for sustainable urban transport solutions. *Eng. Appl. Artif. Intel.* 2024;128:107447.
- [8] Badawy A, Fisteus JA, Mahmoud TM, Abd El-Hafeez T. *Topic extraction and interactive knowledge graphs for learning resources.* *Sustainability* 2021;14(1):226.
- [9] Jbairi O, Ouchani FZ, Ghennioui A, Ferfra M, Cherkaoui M. Technical potential appraisal and optimal site screening comparing AHP and fuzzy AHP methods for large-scale CSP plants: a GIS-MCDM approach in Morocco. *Sustainable Energy Technol. Assess.* 2024;68:103877.
- [10] Mostafa G, Mahmoud H, Abd El-Hafeez T, ElAraby E, M.. The power of deep learning in simplifying feature selection for hepatocellular carcinoma: a review. *BMC Med. Inf. Decis. Making* 2024;24(1):287.
- [11] Alharbi A, Alosaimi W, Alyami H, Alouffi B, Almulihi A, Nadeem M, et al. Selection of data analytic techniques by using fuzzy AHP TOPSIS from a healthcare perspective. *BMC Med. Inf. Decis. Making* 2024;24(1):240.
- [12] Farghaly H, Abd El-Hafeez T. A new feature selection method based on frequent and associated itemsets for text classification. *Concurrency Comput. Pract. Exper.* 2022;34(25):e7258.
- [13] Jain S, Jain VK, Mishra S. Fuzzy-AHP based optimal RSU deployment (Fuzzy-AHP-ORD) approach using road and traffic analysis in VANET. *Ad Hoc Netw.* 2024;161:103529.
- [14] Mahmoud TM, Abd-El-Hafeez T, Badawy A. A framework for an E-learning system based on semantic web. *International Journal on Computer Science and Engineering* 2013;5(8):698.
- [15] Ayvaz B, Tatar V, Sağır Z, Pamucar D. An integrated Fine-Kinney risk assessment model utilizing Fuzzy AHP-WASPAS for occupational hazards in the aquaculture sector. *Process Saf. Environ. Prot.* 2024;186:232–51.
- [16] Khairi M, Mahmoud TM, Abd-El-Hafeez T. The effect of rebalancing techniques on the classification performance in cyberbullying datasets. *Neural Comput. & Applic.* 2024;36(3):1049–65.
- [17] Eliwa EHI, El Koshiry AM, Abd El-Hafeez T, Omar A. Optimal gasoline price predictions: Leveraging the ANFIS regression model. *Int. J. Intell. Syst.* 2024;2024(1):8462056.
- [18] Elmessery WM, Habib A, Shams MY, Abd El-Hafeez T, El-Messery TM, Elsayed S, et al. Deep regression analysis for enhanced thermal control in photovoltaic energy systems. *Sci. Rep.* 2024;14(1):30600.
- [19] Mostafa G, Mahmoud H, Abd El-Hafeez T, ElAraby ME. Feature reduction for hepatocellular carcinoma prediction using machine learning algorithms. *Journal of Big Data* 2024;11(1):88.
- [20] Farghaly HM, Ali AA, Abd El-Hafeez T. Building an effective and accurate associative classifier based on support vector machine. *Sylwan* 2020;164(3):39–56.
- [21] Farghaly, H. M., Ali, A. A., & El-Hafeez, T. A. (2020b). Developing an efficient method for automatic threshold detection based on hybrid feature selection approach. In *Artificial Intelligence and Bioinspired Computational Methods: Proceedings of the 9th Computer Science on-Line Conference 2020, Vol. 2* 9 (pp. 56–72). Springer International Publishing.
- [22] Alejandrino C, Mercante IT, Bovea MD. Combining O-LCA and O-LCC to support organizational circular economies: Methodology and case study. *J. Clean. Prod.* 2022;336:130365.
- [23] Goyal S, Garg D, Luthra S. Analyzing critical success factors to adopt sustainable consumption and production linked with circular economy. *Environ. Dev. Sustain.* 2022;24(4):5195–224.
- [24] Y. Shang M, Song X, Zhao The development of China's Circular Economy: from the perspective of environmental regulation. *Waste Management* 2022 pp. 149, 186–198.
- [25] Mukherjee PK, Das B, Bhardwaj PK, Tampha S, Singh HK, Chanu LD, et al. Socio-economic sustainability with circular economy—an alternative approach. *Sci. Total Environ.* 2023;166630.
- [26] Avrami E, Filatov EA, Durmanov AS, Shkalenko AV, Kosov ME, Ponkratov VV, et al. A New Concept of Consumer Behavior in the Circular Economy. *Emerging Science Journal* 2024;8(4):1536–53.
- [27] Delgado A, Rodriguez DJ, Amadei CA, Makino M. Water in circular economy and resilience (WICER) framework. *Util. Policy* 2024;87:101604.
- [28] Khan SA, Mubarik MS, Paul SK. Analyzing cause and effect relationships among drivers and barriers to circular economy implementation in the context of an emerging economy. *J. Clean. Prod.* 2022;364:132618.
- [29] Ramčilović Jesih A, Simić G, Konatar L, Brljak Z, Šprajc P. Energy efficiency as a driver of the circular economy and carbon neutrality in selected countries of Southern Europe: a soft computing approach. *Energy, Sustainability and Society* 2024;14(1):22.
- [30] Yu Y, Xu J, Zhang JZ, Wu Y, Liao Z. Do circular economy practices matter for financial growth? an empirical study in China. *J. Clean. Prod.* 2022;370:133255.
- [31] Nishitani K, Kokubu K, Wu Q, Kitada H, Guenther E, Guenther T. Material flow cost accounting (MFCA) for the circular economy: an empirical study of the triadic relationship between MFCA, environmental performance, and the economic performance of Japanese companies. *J. Environ. Manage.* 2022;p. 303:114219.
- [32] Awino FB, Apitz SE. Solid waste management in the context of the waste hierarchy and circular economy frameworks: an international critical review. *Integr. Environ. Assess. Manag.* 2024;20(1):9–35.
- [33] D'Adamo I, Mazzanti M, Morone P, Rosa P. Assessing the relation between waste management policies and circular economy goals. *Waste Manag.* 2022;154:27–35.
- [34] Kristinayanti WS, Zaika Y, Devia YP, Wibowo MA. Green Construction and Local Wisdom Integration for Sustainability: a Systematic Literature Review. *Civil Engineering Journal* 2024;10(11):3779–802.
- [35] Ogunmakinde OE, Egbelakin T, Sher W. Contributions of the circular economy to the UN sustainable development goals through sustainable construction. *Resour. Conserv. Recycl.* 2022;178:106023.
- [36] Kee SH, Chiongson JBV, Saludes JP, Vigneswari S, Ramakrishna S, Bhupalan K. Bioconversion of agro-industry sourced biowaste into biomaterials via microbial factories – a viable domain of circular economy. *Environ. Pollut.* 2021;271(116311):1–15. <https://doi.org/10.1016/j.envpol.2020.116311>.
- [37] Oluleye BI, Chan DW, Olawumi TO. Barriers to circular economy adoption and concomitant implementation strategies in building construction and demolition waste management: a PRISMA and interpretive structural modeling approach. *Habitat Int.* 2022;126:102615.
- [38] Haque F, Fan C, Lee YY. From waste to value: addressing the relevance of waste recovery to agricultural sector in line with circular economy. *J. Clean. Prod.* 2023; 415:137873.
- [39] Fernando Y, Tseng ML, Aziz N, Ikhsan RB, Wahyuni-TD IS. Waste-to-energy supply chain management on circular economy capability: an empirical study. *Sustain Prod Consumption* 2022;31:26–38.
- [40] Ncube A, Fiorentino G, Panfilo C, De Falco M, Ulgiati S. Circular economy paths in the olive oil industry: a Life Cycle Assessment look into environmental performance and benefits. *Int. J. Life Cycle Assess.* 2024;29(8):1541–61.
- [41] Khajuria A, Atienza VA, Chavanich S, Henning W, Islam I, Kral U, et al. Accelerating circular economy solutions to achieve the 2030 agenda for sustainable development goals. *Circular Economy* 2022;1(1):100001.
- [42] Wang JX, Burke H, Zhang A. Overcoming barriers to circular product design. *Int. J. Prod. Econ.* 2022;p. 243:108346.
- [43] Pinheiro MAP, Jugend D, de Sousa L, Jabbour AB, Chiappetta Jabbour CJ, Latan H. Circular economy-based new products and company performance: the role of stakeholders and Industry 4.0 technologies. *Bus. Strateg. Environ.* 2022;31(1): 483–99.
- [44] Gedvilaitė D, Ginevičius R. Assessment of the technological efficiency of production and consumption of EU countries in the context of circular economy. *Journal of International Studies* 2024;17(1).
- [45] Cappelletti F, Rossi M, Germani M. How de-manufacturing supports circular economy linking design and EoL-a literature review. *J. Manuf. Syst.* 2022;63: 118–33.
- [46] Aguiar MF, Jugend D. Circular product design maturity matrix: a guideline to evaluate new product development in light of the circular economy transition. *J. Clean. Prod.* 2022;365:132732.
- [47] Eisenreich A, Füller J, Stuchey M, Gimenez-Jimenez D. Toward a circular value chain: Impact of the circular economy on a company's value chain processes. *J. Clean. Prod.* 2022;378:134375.
- [48] Reslan M, Last N, Mathur N, Morris KC, Ferrero V. Circular economy: a product life cycle perspective on engineering and manufacturing practices. *Procedia CIRP* 2022;105:851–8.
- [49] Hariyani D, Hariyani P, Mishra S, Sharma MK. Leveraging digital technologies for advancing circular economy practices and enhancing life cycle analysis: a systematic literature review. *Waste Management. Bulletin* 2024.
- [50] Alcayaga A, Hansen EG. Smart circular economy as a service business model: an activity system framework and research agenda. *R&D Manag.* 2024.
- [51] Yin S, Jia F, Chen L, Wang Q. Circular economy practices and sustainable performance: a meta-analysis. *Resour. Conserv. Recycl.* 2023;190:106838.
- [52] Sarja M, Onkila T, M'akela, M... A systematic literature review of the transition to the circular economy in business organizations: obstacles, catalysts and ambivalences. *J. Clean. Prod.* 2021;286:125492.
- [53] Kanzari A, Rasmussen J, Nehler H, Ingelsson F. How financial performance is addressed in light of the transition to circular business models-a systematic literature review. *J. Clean. Prod.* 2022;376:134134.
- [54] Försterling G, Orth R, Gellert B. Transition to a circular economy in Europe through new business models: Barriers, drivers, and policy making. *Sustainability* 2023;15(10):8212.
- [55] Nikolakis N, Catti P, Chaloulos A, van de Kamp W, Coy MP, Alexopoulos K. A methodology to assess circular economy strategies for sustainable manufacturing using process eco-efficiency. *J. Clean. Prod.* 2024;445:141289.

- [56] Dainelli F, Daddi T, Marrucci L. Financial sustainability of circular innovations in SMEs. a case study from the fashion industry in Italy. *J. Clean. Prod.* 2024;451:142042.
- [57] Taghipour A, Akkaltham W, Eaknarajindawat N, Stefanakis AI. The impact of government policies and steel recycling companies' performance on sustainable management in a circular economy. *Resour. Policy* 2022;77:102663.
- [58] Barbillon G, Humbert C, Clavaguera C, Gayral A, Martínez L, Huttel Y, et al. Addressing molecular geometry on dense gold nanoparticle substrates by two-color sum-frequency generation spectroscopy combined with DFT calculations. *Appl. Surf. Sci.* 2025;683:161817.
- [59] Firlı A, Rahadian D, Dinçer H, Yüksel S, Isyuwardhana D, Fariska P. Expert-based sustainable investment strategies for SMEs with hybrid molecular fuzzy learning algorithms. *Sustainable Futures* 2025;100575.
- [60] Dinçer H, Yüksel S, Olaru GO, Eti S. Integrated information system based on Q-learning algorithm and multi-objective particle swarm optimization with molecular fuzzy-based decision-making for corporate environmental investments. *Inf. Sci.* 2025;698:121757.
- [61] Celebre G. Might molecular symmetry operations exist "partially"? *J. Math. Chem.* 2024;62(2):406–24.
- [62] Kou G, Dinçer H, Ergün E, Eti S, Yüksel S, Hacıoğlu Ü. Enhancing decentralized energy storage investments with artificial intelligence-driven decision models. *Artif. Intell. Rev.* 2025;58(7):1–35.
- [63] Gentili PL. Chemical AI in the Limelight: the Contribution of Photochromic Materials and Oscillatory Chemical Reactions. *Adv. Opt. Mater.* 2025;2500016.
- [64] Rabee AR, Gerges M, Galal EE, Nasim SM, Zakaria A, Hagar M. Photochromic spiro-indolium iodide derivatives; synthesis, Molecular geometry DFT assessment and UV-vis photophysical investigation. *J. Mol. Struct.* 2025;141740.
- [65] Shen Y, Liu W, Dinçer H, Yüksel S. Hybrid molecular fuzzy recommender systems for impact of climate change on renewable energy performance. *Renew. Energy* 2025;245:122799.
- [66] Shen Y, Liu W, Yüksel S, Dinçer H. A molecular fuzzy decision-making model for optimizing renewable energy investments towards carbon neutrality. *Renew. Energy* 2025;240:122175.
- [67] Kumbure MM, Tarkiainen A, Stoklasa J, Luukka P, Jantunen A. Causal maps in the analysis and unsupervised assessment of the development of expert knowledge: Quantification of the learning effects for knowledge management purposes. *Expert Syst. Appl.* 2024;236:121232.
- [68] Gentili PL. The Conformational Contribution to Molecular Complexity and its Implications for Information Processing in living Beings and Chemical Artificial Intelligence. *Biomimetics* 2024;9(2):121.
- [69] Kwon Y, Lee S, Bae J, Park S, Moon H, Lee T, et al. Evaluation of Incinerator Performance and Policy Framework for Effective Waste Management and Energy Recovery: a Case Study of South Korea. *Sustainability* 2024;16(1):448.
- [70] Iqbal A, Yasar A, Nizami AS, Haider R, Sultan IA, Kedwaii AA, et al. Empirical analysis of cost-effective and equitable solid waste management systems: Environmental and economic perspectives. *Environ. Res.* 2024;244:117858.
- [71] Shukla KA, Sofian ADABA, Singh A, Chen WH, Show PL, Chan YJ. Food waste management and sustainable waste to energy: current efforts, anaerobic digestion, incinerator and hydrothermal carbonization with a focus in Malaysia. *J. Clean. Prod.* 2024;448:141457.
- [72] Rafiquee A, Shabbiruddin.. Optimal selection and challenges of municipal waste management system using an integrated approach: a case study. *Energy Sources Part A* 2024;46(1):1996–2023.
- [73] Chen W, Alharthi M, Zhang J, Khan I. The need for energy efficiency and economic prosperity in a sustainable environment. *Gondw. Res.* 2024;127:22–35.
- [74] Jin X, Ahmed Z, Pata UK, Kartal MT, Erdogan S. Do investments in green energy, energy efficiency, and nuclear energy R&D improve the load capacity factor? An Augmented ARDL Approach *Geoscience Frontiers* 2024;15(4):101646.
- [75] Meng L, Li M, Yang H. Enhancing energy efficiency in distributed systems with hybrid energy storage. *Energy* 2024;305:132197.
- [76] Zhang X, Li Y, Xiao F, Gao W. Energy efficiency measures towards decarbonizing Japanese residential sector: Techniques, application evidence and future perspectives. *Energy and Buildings*; 2024.



SERHAT YÜKSEL is Associate Professor of finance in İstanbul Medipol University. Before this position, he worked as a senior internal auditor for seven years in Finansbank, İstanbul-Turkey and 1 year in Konya Food and Agriculture University as an assistant professor. Dr. Yüksel has a BS in Business Administration (in English) from Yeditepe University (2006) with full scholarship. He got his master degree from the economics in Boğaziçi University (2008). He also has a PhD in Banking from Marmara University (2015). His research interests lie in energy economics, banking, finance and financial crisis. He has more than 200 scientific articles and some of them are indexed in SSCI, SCI, Scopus and Econlit. Also, he is the editor of some books that will be published by Springer and IGI Global.



HASAN DİNÇER is Professor of finance at İstanbul Medipol University, Faculty of Economics and Administrative Sciences, İstanbul-Turkey. Dr. Dinçer has BAs in Financial Markets and Investment Management from Marmara University. He received his PhD in Finance and Banking with his thesis entitled "The Effect of Changes on the Competitive Strategies of New Service Development in the Banking Sector". He has work experience in the finance industry as a portfolio specialist and his major academic studies focusing on financial instruments, performance evaluation, and economics. He is the executive editor of the International Journal of Finance and Banking Studies (IJFBS) and the founder member of the Society for the Study of Business and Finance (SSBF). He has about 200 scientific articles and some of them is indexed in SSCI, SCI-Expanded and Scopus. In addition to them, he is also editor of many different books published by Springer and IGI Global.



SERKAN ETİ received the B.S. degree in mathematics from Yıldız Technical University, and the master's and Ph.D. degrees from Marmara University, in 2016. His master's dissertation was titled "A Research on Comparison of Models Explaining the Profitability Base on Financial Data." He is currently an Assistant Professor with İstanbul Medipol University. His research interests include data mining, fuzzy logic, and multi-criteria decision-making methods.



Gabriela Oana Olaru is the assistant professor in İstanbul Medipol University. With 19 years of administrative management experience in the higher education sector, she teaches courses and conduct academic studies in the fields of internet, social media, digital marketing, new communication technologies, creative industries, creative cities, social entrepreneurship, creative entrepreneurship, design-oriented thinking, communication and media at associate, undergraduate and graduate levels as a faculty member. She provides consultancy in the field of communication by designing training, seminars and workshops for institutions and organizations in these fields. She took part in the Design Factory supported by the İstanbul Development Agency as an Academic Coordinator and had the opportunity to gain project experience.



Ümit Hacıoğlu is a professor of finance and business strategy at İbn Haldun University. Applied Business, Corporate Finance, and Strategic Management are the main pillars of his work. Pioneering global publishing houses have published his editorial labors as premier reference books. Prof. Hacıoğlu has editorial roles in international journals. He is nowadays managing editor of International Journal of Research in Business and Social Science, International Journal of Business Ecosystem and Strategy, Bussecon Review of Social Sciences and Bussecon Review of Finance & Banking Studies.



Gang Kou is a professor of School of Management and Economics, University of Electronic Science and Technology of China and managing editor of International Journal of Information Technology & Decision Making. Previously, he was a research scientist in Thomson Co., R&D. He received his Ph.D. in Information Technology from the College of Information Science & Technology, Univ. of Nebraska at Omaha; got his Master degree in Dept of Computer Science, Univ. of Nebraska at Omaha; and B.S. degree in Department of Physics, Tsinghua University, Beijing, China. He has published more than eighty papers in various peer-reviewed journals and conferences.