

Improved Estimation of Population Mean Using Exponentially Weighted Moving Averages for the Time Scaled Surveys

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Abstract

This paper we use exponentially weighted moving average (EWMA) statistic is an improved type statistic that used current and previous information to estimate the population parameter. In this article utilizes EWMA statistic to propose a ratio and product estimator for the surveys based on time scale. The proposed estimators consist of current as well as past sample information. The mean square error expressions of the proposed estimators are derived and mathematical conditions are recognized to prove the efficiency of proposed estimators. The results of simulation study it is discovered from that utilization of the past samples information the performance of estimator in expressions of efficiency. Real life examples are obtainable to determine the use of proposed estimators.

Keywords: *Exponentially Weighted Moving Average, Ratio Estimator, Auxiliary Variables, Modified Exponentially Weighted Moving Average, Exponential Ratio-type Estimator, Exponential Product-type Estimator.*

Introduction

The aim of statistical analysis is to produce information about some selected population. Population mean is single of the principal measures of central tendency in about all departments of society including field of medical sciences, agriculture, industry, biological sciences, social sciences, humanities etc. Thus, the estimation of population mean is of great consequence in above fields. The estimation of population mean plays an important role in above-mentioned fields. In this study, we will propose few generalized estimators under exponentially weighted moving averages. Ratio-type estimators are efficient when study variables and auxiliary variables are negatively correlated.

Cochran (1940) suggested by ratio estimator is given by

$$\hat{Y}_r = \frac{\bar{y}}{\bar{x}} \bar{X} \quad (1)$$

Robson (1957) introduced the product type estimator using the auxiliary variable for inverse linear correlation between study and auxiliary variable. Where \bar{x} and \bar{y} are the individual sample means of auxiliary variable and study variable. The product estimator is defined as

$$\hat{Y}_p = \frac{\bar{y}}{\bar{X}} \bar{x} \quad (2)$$

The mean square error (MSE) expression for ratio and product estimator is given by

$$MSE(\hat{Y}_r) \approx \theta[C_y^2 + C_x^2 - 2\rho C_y C_x] \quad (3)$$

And

$$MSE(\hat{Y}_p) \approx \theta[C_y^2 + C_x^2 + 2\rho C_y C_x] \quad (4)$$

Existing Memory Type Estimator

The EWMA statistic was (Roberts, 1959) by the first used for the purpose to make use of previous and present information at the same time to increase the efficiency of the estimators are following.

The EWMA statistic suggested by Roberts, (1959) is given by

$$Z_i = \lambda \bar{y} + (1 - \lambda)Z_{i-1} \quad (5)$$

And

$$Q_i = \lambda \bar{x} + (1 - \lambda)Q_{i-1} \quad (6)$$

The expected value and variance of exponentially weighted moving average is given by

$$E(z_i) = \mu_y$$

Where \bar{y}_t the sample is mean at time t , λ constant of observations is the weight parameter. Its value ranges from $0 \leq \lambda \leq 1$. As λ moves from 0 to 1 present information obtain further weight and at the same time previous information goes down its weight.

$$\text{var}(z_i) = \frac{\sigma_y^2}{n} \left[\frac{\lambda}{2 - \lambda} \left(1 - (1 - \lambda)^{2i} \right) \right] \quad (7)$$

The variance limiting expression of has defined as:

$$\text{var}(z_i) = \frac{\sigma_y^2}{n} \left[\frac{\lambda}{2 - \lambda} \right] \quad (8)$$

Using Z and Q the existing memory type ratio and product estimators are formulated as:

$$\bar{Y}_{rmi} = \frac{Z_i}{Q_i} \mu_x \quad (9)$$

And

$$\bar{Y}_{pmi} = \frac{Z_i}{\mu_x} Q_i \tag{10}$$

The mean square error memory-type ratio estimator is defined as

$$MSE(\hat{Y}_{rmi}) \approx \theta \left[\frac{Var(Z_i)}{\bar{Y}^2} + \frac{Var(Q_i)}{\bar{X}^2} - 2 \frac{Cov(Z_i, Q_i)}{\bar{Y}\bar{X}} \right] \tag{11}$$

$$MSE(\hat{Y}_{rmi}) \approx \theta \frac{\lambda}{2-\lambda} [C_y + C_x - 2\rho C_y C_x] \tag{12}$$

The mean square error for the proposed product-type estimator:

$$MSE(\hat{Y}_{pmi}) \approx \theta \frac{\lambda}{2-\lambda} [C_y + C_x + 2\rho C_y C_x] \tag{13}$$

We derive the proposed estimator on the base of above-mentioned estimators and by using numerical databases or simulation study; we justify the efficiency of proposed estimators with existing estimators.

Where μ_x population is mean of auxiliary variable and supposed to be known in advance. The mean square error expression of the estimator is derived by Taylor series approximation using the following term Approximation by Taylor series using the following expression:

Let

$$e_y = \frac{Z_i - \mu_y}{\mu_y}; e_x = \frac{Q_i - \mu_x}{\mu_x}; e_0 = \frac{Z_i - \mu_y}{\mu_y}; e_1 = \frac{\mu_x - Q_i}{\mu_x}; e_2 = \frac{\mu_R - T_i}{\mu_R}; \text{ and } e_3 = \frac{s_{yx} - S_{yx}}{S_{yx}}$$

$$\lambda = \frac{\alpha \bar{X}}{\alpha \bar{X} + \beta}, t = 1$$

Such that $E(e_y) = E(e_x) = E(e_0) = E(e_1) = E(e_2) = E(e_3) = 0$ for $i=0, 1, 2,$ and 3

$$E(e_0^2) = \theta C_y^2 = \theta \frac{Var(Z_i)}{\bar{Y}^2} = \theta \frac{\sigma^2}{\bar{Y}^2} \left[\frac{\lambda}{2-\lambda} \right]$$

$$E(e_1^2) = \theta C_x^2 = \theta \frac{Var(Q_i)}{\bar{X}^2} = \theta \frac{\sigma^2}{\bar{X}^2} \left[\frac{\lambda}{2-\lambda} \right]$$

$$E(e_2^2) = \theta C_R^2 \quad ; \quad E(e_3^2) = \theta \left(\frac{\theta_{22x}}{\rho_{yx}} - 1 \right) \quad ;$$

$$E(e_0 e_1) = \theta \rho_{xy} C_x C_y = \theta C_{yx} = \theta \frac{Cov(Z_i, Q_i)}{\bar{Y}\bar{X}} = \theta \rho C_y C_x \left[\frac{\lambda}{2-\lambda} \right]$$

$$E(e_1e_2) = \theta \rho_{xR} C_x C_R = \theta C_{xR} ; \quad E(e_0e_2) = \theta \rho_{yR} C_y C_R = \theta C_{yR} ; \quad E(e_1e_3) = \theta \left(\frac{C_x \theta_{12x}}{\rho_{yx}} \right) ;$$

$$E(e_2e_3) = \theta \left(\frac{C_r \theta_{12r}}{\rho_{yr}} \right)$$

Finite population correction factor $\theta = \left(\frac{1}{n} - \frac{1}{N} \right)$

Estimates related to study variable, auxiliary variable and the ranks of auxiliary variable. R is the rank of auxiliary variables and population mean are as following

$$\bar{Y} = N^{-1} \sum_{i=1}^N y_i \quad \bar{X} = N^{-1} \sum_{i=1}^N x_i \quad \bar{R} = N^{-1} \sum_{i=1}^N r_i$$

Sample mean $\bar{y} = n^{-1} \sum_{i=1}^n y_i \quad \bar{x} = n^{-1} \sum_{i=1}^n x_i \quad \bar{r} = n^{-1} \sum_{i=1}^n r_i$

Population variance $S_y^2 = \frac{\sum_{i=1}^N (y_i - \bar{Y})^2}{(N-1)} \quad S_x^2 = \frac{\sum_{i=1}^N (x_i - \bar{X})^2}{(N-1)} \quad S_r^2 = \frac{\sum_{i=1}^N (r_i - \bar{R})^2}{(N-1)}$

Proposed Improved Estimation Of Population Mean Using Ewma For The Time Scaled Surveys

A weighted average of all previous and present observations has known as the exponentially weighted moving average (EWMA). The configuration of control chart of exponentially weighted moving average for observing procedure mean and dispersion many innovations and design are established. To estimate the population parameter as it is a memory-type statistic the exponentially weighted moving average statistic used present and past information.

$$\bar{y}_{Hi} = \left[k_2 Z_i + k_3 (\mu_x - Q_i) + k_4 (\mu_R - T_i) \right] \exp \left(\frac{\alpha (\mu_x - Q_i)}{\alpha (\mu_x + Q_i) + 2\beta} \right) \tag{14}$$

Population mean under simple random sampling for efficient estimation of work based on the use of real auxiliary information. Use of auxiliary information to research the biased estimators for improved estimation of population mean using exponentially weighted moving averages for the time scale surveys under simple random sampling. Used of additional information of the ranked auxiliary variables to proposed EWMA efficient estimators under simple and stratified random sampling. We initiated a blend of concepts to explore a EWMA estimators for estimating the improved estimation of population mean.

Proposed improved Ratio estimators are as follows

$$\bar{y}_{rHi} = \left[k_2 Z_i + k_3 (\mu_x - Q_i) + k_4 (\mu_R - T_i) \right] \exp \left(\frac{\alpha (\mu_x - Q_i)}{\alpha (\mu_x + Q_i) + 2\beta} \right)$$

(15)

$$Bias(\bar{y}_{rHi}) \cong -\mu_y + \frac{1}{2} \theta \lambda C_x \{k_3 \mu_x C_x + k_4 \mu_R C_R \rho_{xR}\} + k_2 \mu_y \left\{ 1 + \theta \lambda C_x \left(\frac{3}{8} \lambda C_x - \frac{1}{2} \frac{C_{yx}}{C_x} \right) \right\} \quad (16)$$

Check whether improved estimator of (\bar{y}_{rHi}) is variance

$$\bar{y}_{rHi} = [k_2 \mu_y (1 + e_0) + k_3 \mu_x - \mu_x (1 + e_1) + k_4 \mu - \mu_R (1 + e_2)] \exp\left(\frac{\alpha(\mu_x - \mu_x(1 + e_1))}{\alpha \mu_x + \mu_x(1 + e_1) + 2\beta}\right) + \mu_y - \frac{1}{2} \theta \lambda C_x \{k_3 \mu_x C_x + k_4 \mu_R C_R \rho_{xR}\} - k_2 \mu_y \left\{ 1 + \theta \lambda C_x \left(\frac{3}{8} \lambda C_x - \frac{1}{2} \frac{S_{yx}}{S_x} \right) \right\} \quad (17)$$

After simplification and replacing the parameter μ_y and Z_i biased estimator μ_x and Q_i in equation (18), the proposed estimator is as follows

$$(\bar{y}_{rHi} - \mu_y) = \bar{y}_{rHi} + Z_i(1 - k_2) - \frac{1}{2} \theta \lambda \left(k_3 C_x^2 Q_i - k_4 C_{xR} T_i - \frac{3}{4} k_2 \lambda C_x^2 Z_i - k_2 \frac{S_{yx}}{S_x} \right) \quad (18)$$

Subtracting μ_y on both sides equation (18), we get

$$(\bar{y}_{rHi} - \mu_y) \cong \mu_y \left(1 - \frac{3}{8} k_2 \theta \lambda^2 C_x^2 \right) e_0 - \left(\frac{1}{2} k_2 \mu_y \lambda + k_3 \mu_x \right) e_1 - k_4 \mu_R e_2 + \frac{1}{2} k_2 \theta \lambda \frac{S_{yx}}{\mu_x} e_3 - \frac{1}{2} k_2 \mu_y \lambda e_0 e_1 + \frac{1}{2} k_4 \mu_R \lambda e_1 e_2 + \left(\frac{3}{8} k_2 \mu_y \lambda^2 + \frac{1}{2} k_3 \mu_x \lambda \right) e_1^2 - \frac{1}{2} \theta \lambda \left(k_3 \mu_x C_x^2 + k_4 \mu_R C_{xR} + \frac{3}{4} k_2 \mu_y \lambda C_x^2 - k_2 \mu_y C_{yx} \right)$$

Taking expectation on both sides, we have the $Bias(\bar{y}_{rHi})$

$$Bias(\bar{y}_{rHi}) = E(\bar{y}_{rHi} - \mu_y) = -\frac{1}{2} k_2 \mu_y \lambda \theta C_{yx} + \frac{1}{2} k_4 \mu_R \lambda \theta C_{Rx} + \frac{3}{8} k_2 \mu_y \lambda^2 \theta C_x^2 + \frac{1}{2} k_3 \mu_x \lambda \theta C_x^2 + \frac{1}{2} k_2 \mu_y \lambda \theta C_{yx} - \frac{1}{2} k_4 \mu_R \lambda \theta C_{Rx} - \frac{3}{8} k_2 \mu_y \lambda^2 \theta C_x^2 - \frac{1}{2} k_3 \mu_x \lambda \theta C_x^2 \quad (19)$$

All terms are canceled out and we have zero bias which shows that the proposed ratio estimator is unbiased. As the first order approximation is used in deriving the expression therefore the term “ratio estimator” is added here, so

$$Bias(\bar{y}_{rHi}) = E(\bar{y}_{rHi} - \mu_y) \cong 0$$

$$(\bar{y}_{rHi} - \mu_y) \cong \mu_y \left(1 - \frac{3}{8} k_2 \theta \lambda^2 C_x^2 \right) e_0 - \left(\frac{1}{2} k_2 \mu_y \lambda + k_3 \mu_x \right) e_1 - k_4 \mu_R e_2 + \frac{1}{2} k_2 \theta \lambda \frac{S_{yx}}{\mu_x} e_3 - \frac{1}{2} k_2 \mu_y \lambda e_0 e_1 + \frac{1}{2} k_4 \mu_R \lambda e_1 e_2 + \left(\frac{3}{8} k_2 \mu_y \lambda^2 + \frac{1}{2} k_3 \mu_x \lambda \right) e_1^2 - \frac{1}{2} \theta \lambda \left(k_3 \mu_x C_x^2 + k_4 \mu_R C_{xR} + \frac{3}{4} k_2 \mu_y \lambda C_x^2 - k_2 \mu_y C_{yx} \right) \quad (20)$$

Now, gain the terms of variance of the proposed estimator taking square and expectations both sides of equation (20) and ignoring the higher term of ℓ we achieve the mean square error of proposed estimators up to first order of approximation as

In order to obtain the values of $\theta_{21x}, \theta_{12x}, \theta_{22x}$ and θ_{12r} following expression are used

Now, μ_{abR} can be calculated as:

$$\theta_{abx} = \frac{\mu_{abx}}{\left(\frac{a}{\mu_{20x}^2}\right)\left(\frac{b}{\mu_{02x}^2}\right)}; \theta_{abR} = \frac{\mu_{abR}}{\left(\frac{a}{\mu_{20R}^2}\right)\left(\frac{b}{\mu_{02R}^2}\right)}$$

While a=b=0, 1, and 2

$$\mu_{abx} = \frac{\sum_{i=1}^N (z_i - \bar{Y})^a (Q_i - \bar{X})^b}{N}; \mu_{abR} = \frac{\sum_{i=1}^N (z_i - \bar{Y})^a (T_i - \bar{R})^b}{N}$$

$$A_1 = \frac{3}{4} \phi \lambda C_y^2 C_x^2 + C_{yx} - \frac{\phi C_{yx} C_y \theta_{21x}}{\rho_{yx}}; \quad ;$$

$$A_2 = \frac{1}{4} C_x^2 - \frac{9}{64} \phi \lambda^2 C_x^4 - \frac{1}{4} \phi C_{yx}^2 - \frac{\phi C_{yx} C_x \theta_{12x}}{2\rho_{yx}} + \frac{3}{4} \phi \lambda C_x^2 C_{yx}$$

$$A_3 = 1 - \frac{1}{4} \phi \lambda^2 C_x^2; \quad A_4 = 1 - \frac{1}{4} \phi \lambda^2 C_x^2 \rho_{xR}^2$$

$$A_5 = C_x + \frac{5}{4} \phi \lambda C_x C_{yx} - \frac{\phi C_{yx} \theta_{12x}}{\rho_{yx}} - \frac{3}{8} \phi \lambda^2 C_x^3$$

$$A_6 = \frac{3}{4} \phi \lambda C_x^2 C_{yx} + C_{xR} - \frac{\phi C_{yx} C_R \theta_{12R}}{\rho_{yR}} + \frac{1}{2} \phi \lambda C R C_{yx} - \frac{3}{8} \phi \lambda^2 C_x^2 C_{xR}$$

In order to obtain the values of $A_1, A_2, A_3, A_4, A_5, A_6$ following expression are using:

$$\begin{aligned} V(\bar{y}_{rHi}) &\cong E(\bar{y}_{rHi} - \mu_y)^2 = \mu_y^2 \theta C_y^2 - \mu_y^2 \lambda \theta A_1 k_2 - \mu_x \mu_y \theta C_{yx} k_3 - 2\mu_R \mu_y \theta C_{Ry} k_4 \\ &+ \mu_y^2 \lambda^2 \theta A_2 k_2^2 + \mu_x^2 \theta C_x^2 A_3 k_3^2 + \mu_R^2 \theta C_R^2 A_4 k_4^2 + \mu_x \mu_y \theta \lambda C_x A_5 k_3 k_2 + \mu_R \mu_y \theta \lambda A_6 k_2 k_4 \\ &+ 2\mu_R \mu_x \theta C_{xR} A_3 k_3 k_4 \end{aligned} \quad (21)$$

Partially differentiating equation (23) with respect to K_2, K_3 and K_4 equating them to zero. We have, values of K_2, K_3 and K_4 as given by

$$K_2 = \frac{2C_1}{\lambda C_2} ; \quad K_3 = \frac{\mu_y [A_1 B_2 C_2 - 4A_2 B_2 C_1 - A_6 (B_3 C_2 - B_6 C_1)]}{\mu_x C_x A_5 B_2 C_2} ;$$

$$K_4 = \frac{\mu_y [B_3 C_2 - B_6 C_1]}{\mu_R}$$

Placing these values in equation (21), we obtained the minimum variance as defined by

$$\text{Where, } B_1 = C_x A_1 A_3 - C_{yx} A_5 ; \quad B_2 = C_{xR} A_3 A_6 - C_R^2 C_x A_4 A_5 ; \quad B_3 = C_{xR} A_1 A_3 - C_{yR} C_x A_5 ;$$

$$B_4 = C_x A_3 A_6 - C_{xR} A_3 A_5$$

$$B_5 = C_x (4A_2 A_3 - A_5^2) ; \quad B_6 = 4C_{xR} A_2 A_3 - C_x A_5 A_6 ; \quad C_1 = B_1 B_2 - B_3 B_4 ;$$

$$C_2 = B_2 B_5 - B_4 B_6$$

$$D_1 = A_1 B_2 C_2 - 4A_2 B_2 C_1 - A_6 (B_3 C_2 - B_6 C_1) ; \quad D_2 = (B_3 C_2 - B_6 C_1)$$

$$V_{\min}(\bar{y}_{tHt}) \cong \frac{1}{C_x A_5^2 B_2^2 C_2^2} \mu_y \theta \left[C_x A_5^2 B_2^2 \{ C_y^2 C_2^2 - 2A_1 C_1 C_2 + 4A_2 C_1^2 \} + D_1 \{ C_x A_3 D_1 - 2C_{yx} A_5 B_2 C_2 + 2C_x A_5^2 B_2 C_1 \} \right. \\ \left. + C_x A_5^2 D_2 \{ C_R^2 A_4 D_2 - 2C_{yR} B_2 C_2 + 2A_6 B_2 C_1 \} + 2C_{xR} A_3 A_5 D_1 D_2 \right] \quad (22)$$

Proposed Modified Ewma Using Exponential Ratio-Type Estimator

Let $Y_i, i=1, 2, \dots$, be order of independent and identically distributed (*iid*) observations from a normal process with mean and variance equal to μ and σ^2 we study the process to be in-control when $\mu = \mu_0$, while it is out-of-control when $\mu = \mu_1 \neq \mu_0$. The MEWMA statistic at time t or i is defined as

$$MZ_i = \bar{y}_i \lambda + (1 - \lambda) Z_{i-1} + K(\bar{y}_i - \bar{y}_{i-1}) \quad (23)$$

$$E(Z_i) = \mu_y$$

Where $0 \leq \lambda \leq 1$ is the smoothing parameter, K is an added parameter and $Z_0 = \bar{y}_0 = \mu_0$ are the initial values. The expression (1) is linked to the error between the current two observations. The statistic Z_i can also be written

$$MZ_i = (\lambda + K)[\bar{y}_i + (1 - \lambda)\bar{y}_{i-1}] - K[\bar{y}_{i-1} + (1 - \lambda)\bar{y}_{i-2}] + (1 - \lambda)^2 [(\lambda + K)\bar{y}_{i-2} - K\bar{y}_{i-3} + (1 - \lambda)] Z_{i-3}$$

$$MZ_i = (\lambda + K) \sum_{j=1}^i (1 - \lambda)^{i-j} \bar{y}_j - K \sum_{j=1}^{i-1} (1 - \lambda)^{i-j-1} \bar{y}_j + (1 - \lambda - K)(1 - \lambda)^{i-1} \mu_0 \quad (24)$$

The mean value is equal to μ_y and its variance is defined as

$$Var(MZ_i) = \left[\frac{(\lambda + 2\lambda K + 2K^2) - \lambda(1 - \lambda - K)^2 (1 - \lambda)^{2(i-1)}}{(2 - \lambda)} \right] \sigma^2$$

After simplification the time varying control limits (UCL_t and LCL_t) the central limit (CL) of the MEWMA control chart are defined as

$$\begin{aligned}
 UCL_t &= \mu_0 + L\sigma \sqrt{\frac{(\lambda + 2\lambda K + 2K^2) - \lambda(1 - \lambda - K)^2(1 - \lambda)^{2(t-1)}}{(2 - \lambda)}} \\
 CL &= \mu_0 \\
 LCL_t &= \mu_0 - L\sigma \sqrt{\frac{(\lambda + 2\lambda K + 2K^2) - \lambda(1 - \lambda - K)^2(1 - \lambda)^{2(t-1)}}{(2 - \lambda)}}
 \end{aligned}
 \tag{25}$$

Where $L > 0$ is the width of control limits. For large values of t and as

$\lim_{t \rightarrow \infty} \lambda(1 - \lambda - K)^2(1 - \lambda)^{2(t-1)} = 0$, the variance of the statistic Z_i is about equal to

$$\text{Var}(MZ_t) = \frac{(\lambda + 2\lambda K + 2K^2)}{(2 - \lambda)}$$

And the asymptotic control limits of the MEWMA chart are developed as

$$\begin{aligned}
 UCL_t &= \mu_0 + L\sigma \sqrt{\frac{(\lambda + 2\lambda K + 2K^2)}{(2 - \lambda)}} \\
 CL &= \mu_0 \\
 LCL_t &= \mu_0 - L\sigma \sqrt{\frac{(\lambda + 2\lambda K + 2K^2)}{(2 - \lambda)}}
 \end{aligned}
 \tag{26}$$

We notice that the asymptotic variance of MEWMA chart is reduced for $K = -\frac{\lambda}{2}$. The MEWMA chart

is made by plotting the statistic Z_i against the sample number or time t . A process is measured to be out-of-control if any plotting statistic goes above the control limits. The procedure is measured to be in-control. Opinion out that the MEWMA chart moderates to the classical EWMA chart when $K = 0, 1$. We suggested exponentially weighted moving average (EWMA) control charts using exponential ratio-type estimators, EWMA control charts using these ratio estimators are established to observe process mean by finding specific ARLs being appropriate when small shifts are of interest. Proposed MEWMA Control Charts Using Exponential Ratio-Type Estimators the assumed study variable Y is problematic to measure straight but it is easy to measure with the consistent auxiliary variable X . Let \bar{y}_{it} be the sample mean under simple random sampling subsequent of \bar{x}_{it} at time $t(t = 1, 2, 3, 4, \dots)$; exponential ratio-type estimator for mean under SRS, are defend as

$$\hat{Y}_{ri} = Z_i \left[\exp \frac{(Q_i - \mu_x)}{(Q_i + \mu_x)} \right]
 \tag{27}$$

With mean $\mu_y = E(\bar{y}_{ri}) = \bar{Y}$; $\mu_x = E(\bar{x}_{ri}) = \bar{X}$

Where constant $\therefore C = \rho(C_y, C_x)$

$$Var(\hat{Y}_{ri}) = \theta\mu_y^2 \left[\frac{\lambda}{2-\lambda} \right] \left[C_y^2 + \frac{1}{4} C_x^2 (1-C) \right] \quad (28)$$

The variance of the statistic of MEWMA control chart using exponential ratio-type estimator are defined as

$$\begin{aligned} Var(\hat{Y}_{ri}) &= \theta\mu_y^2 \left[\frac{\lambda}{2-\lambda} \right] \left[C_y^2 + \frac{1}{4} C_x^2 (1-C) \right] - Var(MZ_t) \\ Var(\hat{Y}_{ri}) &= \theta\mu_y^2 \left[\frac{\lambda}{2-\lambda} \right] \left[C_y^2 + \frac{1}{4} C_x^2 (1-C) \right] - \frac{(\lambda + 2\lambda K + 2K^2)}{(2-\lambda)} \end{aligned} \quad (29)$$

The modified exponentially weighted moving averages control charts using exponential product type estimator are given by

$$\begin{aligned} \hat{Y}_{pi} &= Z_t \exp \left[\gamma \left(\frac{Q_i - \mu_x}{Q_i + (a-1)\mu_x} \right) \right] \\ E(Z_t) &= \bar{Y} = \mu_y \end{aligned} \quad (30)$$

Where constant

$$\begin{aligned} \therefore C &= \rho(C_y / C_x) \\ Var(\hat{Y}_{pi}) &= \theta\mu_y^2 \left[\frac{\lambda}{2-\lambda} \right] (C_y^2 - C^2 C_x^2) \end{aligned} \quad (31)$$

For large no of times, the variance of $Var(Z_t)$ becomes and variance of the statistics of MEWMA control chart using exponential ratio-type estimators are defined as

$$Var(\hat{Y}_{pi}) = \theta\mu_y \left[\frac{\lambda}{2-\lambda} \right] (C_y^2 - C^2 C_x^2) - \frac{(\lambda + 2\lambda K + 2K^2)}{(2-\lambda)} \quad (32)$$

Control limits for the proposed MEWMA control charts, L_1 and L_2 are the constant multipliers. The values of L_1 and L_2 are select such that the in-control ARLs of the proposed MEWMA control chart using exponential ratio-type estimator reach a specific smooth of decided value of target ARL of the procedure.

$$\begin{aligned} UCL_t &= \mu_y + L_1 \delta Var(MZ_t) \\ LCL_t &= \mu_y + L_1 \sigma Var(MZ_t) \\ CL_t &= \mu_y \end{aligned} \quad (33)$$

$$UCL_t = \mu_y + L_2 \delta \text{Var}(MZ_t)$$

$$LCL_t = \mu_y + L_2 \sigma \text{Var}(MZ_t)$$

Now the probability of recording the procedure as out of control when truly the process is in control is found as

$$P = P(MZ_t < LCL_1) + P(MZ_t > UCL_1)$$

$$P = \left[\frac{Z_t - \mu_y}{\sigma} < \frac{LCL_1 - \mu_y}{\sigma} \right] + P \left[\frac{Z_t - \mu_y}{\sigma} > \frac{UCL_1 - \mu_y}{\sigma} \right] \quad (34)$$

The term $\phi(\cdot)$ is cumulative distribution function of the standard normal distribution

$$P = \phi(-L_1) + 1 - \phi(L_1)$$

$$P = 2[1 - \phi(L_1)] \quad (35)$$

The probability for the planned control chart is defined as follows

$$P = P\{UCL_2 < Z < UCL_1\} + P\{LCL_1 < Z < LCL_2\}$$

$$P = 2[\phi(L_1) - \phi(L_2)] \quad (36)$$

So that we get,

$$P_0 = \frac{P}{1 - P}; \quad ARL_0 = \frac{1}{P}$$

In shifted process, if our mean is shift

$$\mu_1 = \mu_0 + f\sigma \quad \therefore \mu_0 = \mu_y$$

$$\mu_1 = \mu_y + f\sigma \sim N\left(\mu_y + f\sigma, \frac{\lambda}{2 - \lambda} \text{Var}(MZ_t)\right) \quad (37)$$

Now the probability of the process as out of control when the shift will be introduced in the process mean is defined as

$$P_1 = P(Z_t < LCL_1 | \mu_1) + P(Z_t > UCL_1 | \mu_1)$$

Where, $K = 0, -\frac{\lambda}{2}, -\frac{\lambda}{4}, \frac{\lambda}{2}, \frac{\lambda}{4}, \lambda, 1$

After simplification, the above expression as given by

$$P_1 = \theta \left(L_1 - \frac{f\sigma_y}{sd(Z_t)} \right) - \theta \left(L_2 - \frac{f\sigma_y}{sd(Z_t)} \right) + \theta \left(-L_2 - \frac{f\sigma_y}{sd(Z_t)} \right) - \theta \left(-L_1 - \frac{f\sigma_y}{sd(Z_t)} \right) \quad (38)$$

$$P_1 = \frac{P}{1-P}; \quad ARL_1 = \frac{1}{P} \quad (39)$$

Simulation Study

In this segment the results from simulation study are introduced to judge the computed of proposed estimators. The comparison of the proposed estimator (\hat{y}_{rHi}) is presented With respect to classical ratio estimator (μ_{yr}). The mean square errors and relative Efficiencies are announced built on replications and every replication outing here comprises Fifty thousand replications.

Table 1. MSE of the improved ratio estimators at different values of λ, ρ and n.

ρ	N	$\lambda = 0.05$				$\lambda = 0.1$				$\lambda = 0.25$				$\lambda = 0.5$				$\lambda = 0.75$				$\lambda = 1$	
		\hat{Y}_r	\hat{Y}_{rmi}	\hat{Y}_{rHi}	\hat{Y}_r	\hat{Y}_{rmi}	\hat{Y}_{rHi}	\hat{Y}_r	\hat{Y}_{rmi}	\hat{Y}_{rHi}	\hat{Y}_r	\hat{Y}_{rmi}	\hat{Y}_{rHi}	\hat{Y}_r	\hat{Y}_{rmi}	\hat{Y}_{rHi}	\hat{Y}_r	\hat{Y}_{rmi}	\hat{Y}_{rHi}	\hat{Y}_r	\hat{Y}_{rmi}	\hat{Y}_{rHi}	
0.05	10	0.102	0.002	1.68e-8	0.103	0.005	2.24e-8	0.102	0.015	4.86e-9	0.103	0.034	9.98e-10	0.102	0.061	2.17e-10	0.103	0.103	0				
	20	0.051	0.001	1.67e-8	0.051	0	2.22e-8	0.051	0.007	4.67e-9	0.050	0.016	9.23e-10	0.050	0.030	1.87e-10	0.050	0.050	0				
	30	0.034	0.001	1.67e-8	0.033	0.001	2.21e-8	0.033	0.005	4.61e-9	0.033	0.011	8.98e-10	0.034	0.020	1.85e-10	0.034	0.034	0				
	50	0.020	0.001	1.67e-8	0.020	0.001	2.20e-8	0.020	0.003	4.56e-9	0.020	0.006	8.79e-10	0.020	0.012	1.83e-10	0.020	0.020	0				
	200	0.005	0.001	1.67e-8	0.005	0.000	2.20e-8	0.005	0.001	4.51e-9	0.005	0.001	8.57e-10	0.005	0.003	1.81e-10	0.005	0.005	0				
0.25	10	0.093	0.002	5.22e-8	0.094	0.004	4.04e-8	0.093	0.013	3.74e-8	0.095	0.016	2.51e-09	0.094	0.056	1.95e-10	0.094	0.094	0				
	20	0.046	0.001	5.02e-8	0.047	0.002	4.02e-8	0.047	0.006	6.86e-08	0.047	0.010	2.47e-09	0.047	0.028	1.87e-10	0.047	0.047	0				
	30	0.031	0.001	5.02e-8	0.031	0.001	4.01e-8	0.030	0.004	6.86e-08	0.031	0.006	2.45e-09	0.031	0.019	1.85e-10	0.031	0.031	0				
	50	0.018	0.001	5.02e-8	0.018	0.002	4.01e-8	0.018	0.002	6.86e-08	0.019	0.002	2.44e-09	0.019	0.011	1.83e-10	0.018	0.019	0				
	200	0.004	0.001	5.02e-8	0.004	0.002	4.00e-8	0.004	0.001	6.85e-08	0.005	0.001	2.44e-09	0.005	0.003	1.81e-10	0.005	0.005	0				
0.50	10	0.083	0.002	3.03e-05	0.084	0.004	3.48e-06	0.084	0.012	6.87e-08	0.085	0.014	2.51e-09	0.084	0.002	1.95e-10	0.085	0.085	0				
	20	0.042	0.001	3.13e-05	0.042	0.002	3.50e-06	0.042	0.006	6.86e-08	0.042	0.009	2.47e-09	0.042	0.050	1.87e-10	0.042	0.042	0				
	30	0.028	0.001	3.16e-05	0.028	0.001	3.50e-06	0.028	0.004	6.86e-08	0.028	0.006	2.45e-09	0.028	0.025	1.85e-10	0.028	0.028	0				
	50	0.017	0.001	3.19e-05	0.017	0.001	3.51e-06	0.017	0.002	6.86e-08	0.017	0.001	2.44e-09	0.017	0.017	1.83e-10	0.017	0.017	0				
	200	0.004	0.001	3.22e-05	0.004	0.002	3.51e-06	0.004	0.001	6.85e-08	0.004	0.001	2.43e-09	0.004	0.010	1.81e-10	0.004	0.004	0				
0.75	10	0.075	0.002	3.03e-05	0.075	0.004	3.48e-06	0.074	0.011	6.87e-08	0.074	0.013	2.51e-09	0.074	0.044	1.95e-10	0.074	0.074	0				
	20	0.025	0.001	3.13e-05	0.037	0.002	3.50e-06	0.037	0.005	6.86e-08	0.037	0.008	2.47e-09	0.036	0.022	1.87e-10	0.037	0.037	0				
	30	0.015	0.001	3.16e-05	0.025	0.001	3.51e-06	0.025	0.004	6.86e-08	0.025	0.005	2.45e-09	0.025	0.015	1.85e-10	0.025	0.025	0				
	50	0.004	0.001	3.19e-05	0.015	0.001	3.51e-06	0.015	0.002	6.86e-08	0.015	0.001	2.44e-09	0.015	0.009	1.83e-10	0.015	0.015	0				
	200	0.001	0.001	3.22e-05	0.004	0.001	3.51e-06	0.004	0.001	6.85e-08	0.004	0.000	2.43e-09	0.004	0.002	1.81e-10	0.004	0.004	0				
0.95	10	0.066	0.001	3.23e-05	0.001	0.002	3.52e-06	0.001	0.000	6.86e-08	0.001	0.022	2.43e-09	0.001	0.001	1.80e-10	0.001	0.001	0				
	20	0.033	0.001	3.13e-05	0.033	0.002	3.50e-06	0.033	0.005	6.86e-08	0.033	0.007	2.47e-09	0.033	0.020	1.87e-10	0.034	0.034	0				
	30	0.022	0.001	3.16e-05	0.022	0.001	3.51e-06	0.022	0.004	6.86e-08	0.022	0.005	2.45e-09	0.022	0.013	1.85e-10	0.022	0.022	0				
	50	0.013	0.001	3.19e-05	0.013	0.001	3.51e-06	0.013	0.002	6.86e-08	0.013	0.005	2.44e-09	0.013	0.008	1.83e-10	0.013	0.013	0				

	200	0.003	0.001	3.03e-05	0.003	0.000	3.51e-06	.003	0.000	6.85e-08	0.003	0.001	2.43e-09	0.003	0.002	1.81e-10	0.003	0.003	0
	500	0.001	0.001	3.03e-05	0.001	0.000	3.52e-06	0.001	0.000	6.85e-08	0.001	0.000	2.43e-09	0.001	0.001	1.80e-10	0.001	0.001	0

Table 2. RE of the improved ratio estimators at different values of λ, ρ and n.

ρ	n	$\lambda = 0.05$			$\lambda = 0.1$			$\lambda = 0.25$			$\lambda = 0.5$			$\lambda = 0.75$			$\lambda = 1$		
		\hat{Y}_r	\hat{Y}_{rmi}	\bar{y}_{rHi}	\hat{Y}_r	\hat{Y}_{rmi}	\bar{y}_{rHi}	\hat{Y}_r	\hat{Y}_{rmi}	\bar{y}_{rHi}	\hat{Y}_r	\hat{Y}_{rmi}	\bar{y}_{rHi}	\hat{Y}_r	\hat{Y}_{rHi}	\bar{y}_{rHi}	\hat{Y}_r	\hat{Y}_{rmi}	\bar{y}_{rHi}
0.05	10	0.977	38.254	0.003	0.977	18.516	0.004	0.977	6.735	0.009	0.976	2.910	0.001	0.976	1.627	0.0087	0.978	0.978	0.015
	20	0.978	38.320	0.002	0.974	18.785	0.005	0.979	6.896	0.010	0.979	2.950	0.000	0.978	1.633	0.0088	0.976	0.976	0.016
	30	0.979	37.191	0.002	0.980	18.871	0.006	0.981	6.738	0.011	0.977	2.918	0.009	0.979	1.634	0.0089	0.978	0.978	0.018
	50	0.980	38.085	0.001	0.978	19.105	0.007	0.982	6.934	0.012	0.981	2.967	0.008	0.980	1.631	0.0090	0.979	0.976	0.020
	200	0.979	37.327	0.002	0.980	19.281	0.008	0.980	6.792	0.013	0.979	2.940	0.007	0.982	1.636	0.0091	0.979	0.979	0.022
0.25	10	1.064	40.651	0.032	1.063	20.438	0.010	1.061	7.355	0.015	1.059	3.182	0.089	1.059	1.764	0.093	1.064	1.060	0.026
	20	1.066	41.007	0.033	1.064	19.823	0.011	1.060	7.388	0.016	1.060	3.168	0.090	1.063	1.772	0.094	1.061	1.064	0.028
	30	1.061	42.752	0.034	1.060	20.824	0.012	1.063	7.353	0.017	1.064	3.182	0.081	1.064	1.774	0.095	1.064	1.061	0.030
	50	1.620	41.801	0.043	1.064	20.203	0.013	1.060	7.358	0.018	1.060	3.178	0.082	1.059	1.770	0.096	1.054	1.064	0.030
	200	1.067	42.651	0.054	1.064	20.370	0.014	1.062	7.344	0.019	1.064	3.222	0.083	1.064	1.774	0.097	1.063	1.064	0.032
0.50	10	1.189	46.193	0.044	1.191	22.468	0.016	1.189	8.508	0.021	1.184	3.573	0.085	1.186	1.983	0.099	1.190	1.186	0.036
	20	1.345	48.831	0.074	1.187	23.306	0.017	1.188	8.300	0.022	1.189	3.552	0.086	1.189	1.985	0.100	1.185	1.190	0.039
	30	1.349	47.475	0.084	1.193	22.401	0.018	1.189	8.248	0.023	1.190	3.575	0.087	1.190	1.993	0.101	1.189	1.185	0.041
	50	1.350	46.682	0.088	1.190	22.287	0.019	1.187	8.253	0.024	1.191	3.568	0.088	1.189	1.980	0.102	1.193	1.189	0.042
	200	1.348	48.110	0.089	1.189	22.884	0.020	1.191	8.335	0.025	1.189	3.528	0.089	1.191	1.978	0.102	1.190	1.193	0.044
0.75	10	1.353	52.070	0.091	1.351	25.901	0.022	1.343	9.409	0.027	1.342	4.020	0.091	1.345	2.252	0.104	1.345	1.347	0.048
	20	1.506	55.100	0.092	1.352	25.010	0.022	1.349	9.689	0.028	1.348	4.006	0.092	1.347	2.255	0.105	1.348	1.345	0.050
	30	1.509	51.520	0.093	1.349	26.470	0.023	1.359	9.226	0.029	1.350	4.041	0.093	1.350	2.260	0.106	1.352	1.348	0.052
	50	1.513	53.912	0.094	1.351	25.907	0.024	1.349	9.442	0.030	1.347	4.025	0.094	1.350	2.248	0.107	1.351	1.352	0.054
	200	1.512	51.197	0.095	1.349	26.826	0.025	1.350	9.423	0.031	1.351	4.034	0.095	1.349	2.250	0.108	1.351	1.351	0.055
0.95	10	1.515	57.275	0.097	1.504	28.222	0.027	1.503	10.66 1	0.032	1.503	4.498	0.097	1.507	2.535	0.110	1.514	1.505	0.067
	20	1.351	60.981	0.098	1.510	28.392	0.028	1.510	10.53 8	0.033	1.509	4.549	0.098	1.511	2.529	0.111	1.512	1.509	0.069
	30	1.506	62.639	0.099	1.512	29.255	0.029	1.511	10.63 9	0.034	1.512	4.528	0.099	1.512	2.515	0.112	1.517	1.514	0.071
	50	1.513	58.048	0.100	1.513	28.998	0.030	1.516	10.75 8	0.035	1.516	4.532	0.100	1.512	2.518	0.113	1.514	1.512	0.073
	200	1.512	60.801	0.101	1.515	29.615	0.031	1.514	10.58 6	0.036	1.516	4.555	0.101	1.514	2.531	0.114	1.524	1.517	0.075
500	1.515	62.963	0.102	1.516	28.449	0.032	1.516	10.68 2	0.037	1.514	4.568	0.102	1.515	2.527	0.115	1.514	1.514	0.081	

That the proposed improved efficient estimators for estimating population mean it is observably detected from numerical conclusions in situation of SRSWOR. In calculation, this authority is measured concluded a Monte Carlo simulation study using R software. For this article, are used different sample sizes i.e. (n = 10,20,30,50,200 and 500). Following phases are executed to bring out the simulation study:

Phase 1. Choose a simple random sampling without replacement of size n from the populace of size N .

Phase 2. Sample data from use phase 1 to calculate the MSEs and RE of each the existing and proposed estimators.

Phase 3. Phase 1 and 2 are repetitive fifty thousand times.

Phase 4. Find fifty thousand values for MSEs and RE of all estimators.

Phase 5. Fifty thousand values, gained in phase 4 is the MSEs and RE of all estimators.

Remark.

The following term is used for computation of MSEs and RE for each estimator in this article:

$$MSE(\bar{y}_{Hi}) = \frac{1}{50000} \sum_{i=1}^{50000} (t - \mu_y)^2$$

$$R.E(t) = \frac{MSE(\mu_y)}{MSE(t)}$$

Where $t = \bar{y}, \bar{y}_{rHi}, \bar{y}_r, \bar{y}_p, \bar{y}_{pHi}$, from fifty thousand samples is the mean of t .

Where, μ_y or \bar{y} per unit estimator is the mean. The mean square error values are obtainable in Table 1.

Where, the values of relative efficiency are assumed in Table 2.

So, Table 1-2 are presented the values of MSEs and RE comparing the proposed and existing estimators. The time scaled surveys is more efficient as a compare to obtainable estimators for the exponentially weighted moving averages.

Simulation of Proposed Estimator Under Mewma Control Charts

The proposed MEWMA control charts are designed for the drive of monitoring the shifts in the process mean using MEWMA exponential ratio-type estimators under simple random sampling. We used ARLs as a performance condition to compare the performance efficiency of MEWMA control charts. The ARLs tables and graphs are constructed for frequently used values of in-control ARLs, such as 500, for the MEWMA control charts. Different values of the smoothing constant $\lambda (0 \leq \lambda \leq 1)$ are taken with an interval of 0.1 to construct the ARLs and SDRLs table. The zero-state ARL values of the control charts were computed with 100,000 repetitions in R. supposing that the process variance is IC and remains affected. We consider $\mu_0 = 0$ and $\sigma^2 = 1$. The subgroup size was set at $n=1$. However, if subgroups of size $n > 1$ are collected, then simply substitute y_i with the sample mean \bar{y}_i and σ^2 within $\frac{\sigma^2}{n}$ the above equations. The pre-specified value of ARL0 is set to 500 and we select $\lambda = 0.05, 0.10, 0.25,$ and 0.75 for both charts. The value can be selected independently of the value of λ , in this article, we choose the additional parameter $K = 0, -\frac{\lambda}{2}, -\frac{\lambda}{4}, \frac{\lambda}{2}, \frac{\lambda}{4}, \lambda$ and 1. The MEWMA charts with $K = 0$ reducing to

the classical EWMA charts, while the value $K = -\frac{\lambda}{2}$ minimizes the variance of the charting statistic for both modified charts. We use the steady-state control limits to simplify the application and submission of the charts. We point out that only positive shifts (in units of standard deviation) of the process mean are considered in this study because the normal distribution is symmetric, and the results are similar for negative shifts.

Table 3: ARLs And SDRLs Values For The Proposed MEWMA Control Charts Using Exponential Ratio-Type Estimator, When r_0 Is 500.

Application

We presented the examples to the real data determine the application of the proposed estimators.

Table 3: Computation of improved type ratio estimator

\bar{x}	\bar{y}	\bar{Y}_r	Z_i	MZ_i	Q_i	\hat{Y}_{rmi}	\hat{Y}_{rHi}
5973.5	2081	2208.94	2498.93	2469.93	6304.46	2513.46	2.374
5839.9	2118	2300.2	2460.87	2412.74	6258	6258	0.438
5934.6	2327	2485.92	2447.44	2398.52	6225.66	6225.66	1.817
5934.6	2226	2378.8	2425.32	2370.47	6196.56	6196.56	-0.159
6180.3	2465	273596	2449.44	2379.80	6194.93	6194.93	0.292
6255.5	2392	2498.64	2450.98	2381.78	6200.99	6200.99	-0.001
6101.8	2518	2485.66	2445.07	2403.91	6191.07	6191.07	0.025
6097.3	2500	2619.06	2452.39	2419.52	6181.69	6181.69	-1.036
6255.5	2724	2534.29	2457.16	2428.99	6189.07	6189.07	2.088
6378.9	2588	2707.73	2483.82	2454.82	6208.06	6208.06	-1.540
6483.4	2775	2530.78	2494.2	2484.84	6235.59	6235.59	-0.063
6432.8	2438	2735.79	2522.31	2477.22	6255.31	6255.31	-1.154
6402.0	2694	2414.68	2513.86	2512.44	6269.98	6269.98	0.503
6836.2	2592	2499.38	2531.92	2525.33	6326.6	6326.6	-0.878
6913.5	2846	2377.33	2537.92	2532.43	6385.29	6385.29	-1.496
6691.0	2736	2696.99	2568.7	2565.51	6415.86	6415.86	-1.225
6482.9	2855	2676.45	2585.46	2574.21	6422.57	6422.57	-0.449
6511.3	2855	2780	2612.33	2601.83	6431.44	6431.44	-0.947
6778.4	2821	2639.2	2633.25	2537.44	6466.14	6466.14	0.970

Illustrative Example

The dataset is occupied from the information of agricultural Statistics of Pakistan below the ministry of national food security and research for the design of proposed estimator. The area of cultivation respectively and the variable Y and X is well-defined as yield of wheat (in kg). Set the rank of variable X the individual population average values for variable y and x are R gained as $\mu_r = 10.5$, $\mu_y = 2545.4$, $\mu_x = 6341.21$ and $K = \frac{\lambda}{4} = 0.0625$ by taking the average of completely values of sample means. We implement the EWMA and MEWMA charts designed to detect a shifts $\delta = 2.00$. Estimators are achieved as y and x the mean per unit and from the agricultural reports \hat{Y}_r is calculated

using expression (1). The EWMA and MEWMA statistic is calculated from each sample with $\lambda = 0.1$. Table 5 offerings the estimated values of variable an \bar{Y} and \bar{X} , the calculation of proposed estimators.

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